Three Essays on Macroprudential Policy and Learning

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I certify that all material in this thesis which is not my own work has been identified and that no material has previously been submitted and approved for the award of a degree by this or any other University.

Signature: ....

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Abstracts

Chapter 1: *"Bank equity and macroprudential policy"*

This chapter proposes an alternative macroprudential policy in the framework of Gertler, Kiyotaki and Queralto (2012). In their model, the central bank subsidizes bank outside equity, where the subsidy rate is determined by the shadow cost of the deposit. We find that the alternative rule in which the subsidy rate responds to the aggregate bank outside equity ratio is welfare improving because it has a better stabilization effect on the bank asset deterioration after a financial shock. We disentangle different channels through which macroprudential policies affect the economy and demonstrate that the better stabilization in the post-crisis economy has a positive effect on the economy in normal times through security prices.

Published in the Journal of Economic Dynamics and Control, 2016

Chapter 2: *"Stationarity of Econometric Learning with Bounded Memory and* ***a*** *Predicted State Variable"* (Joint work with Tatiana Damjanovic, Sarunas Girdenas)

In this chapter, we consider a model where producers set their prices based on their prediction of the aggregated price level and an exogenous variable, which can be a demand or a cost-push shock. To form their expectations, they use OLS-type econometric learning with bounded memory. We show that the aggregated price follows the random coefficient autoregressive process and we prove that this process is covariance stationary.

Published in Economics Letters, 2015

Chapter 3: "A comment on:"Capital regulation and monetary policy with fragile banks"" (Joint work with Yoske Igarashi)

This chapter comments on Angeloni and Faia (2013, Journal of Monetary Economics), a dynamic stochastic general equilibrium model with a risky banking sector. We identify the sources of inefficiency in the model and disentangle the channels through which banks choose a high level of leverage. We explain that their assumptions that generate banks over-borrowing feature lead to the return on assets and the bankruptcy probability that are unrealistically high. Next, we modify the model by incorporating the banking sector of Gertler and Karadi (2011) into the AF model and show that the calibration result improves.

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Chapter 1

Bank Equity and Macroprudential Policy

Abstract

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Keywords: Macroprudential Policy, Bank Equity, DSGE Model

1.1 Introduction

The financial crisis in 2009 aroused considerable attention to macroprudential policies, which aim at the resilience of the financial system to macroeconomic shocks. Many studies show that individual banks do not internalize the importance of bank capital ratio for the stability of the banking sector,[1] so there is a role that macroprudential policies can play. A variety of macroprudential policies have been proposed in the literature, and most of them suggest a high bank capital ratio in normal times and countercyclical capital requirements.[2] But when the economy is hit by a financial crisis, how should the central bank *quantitatively* adjust the capital requirement over time? This paper addresses this question by using the dynamic stochastic general equilibrium (DSGE) framework developed in Gertler, Kiyotaki and Queralto (2012) (GKQ hereafter). One advantage of their model is that it incorporates financial intermediaries into a quantitative macroeconomic model. In their framework, banks make an endogenous decision on asset holding, leverage, and the liability structure. The model is rich enough to analyze financial propagation quantitatively through bank borrowing constraint and risk-taking behavior, and thereby it enables one to investigate the effect of different macroprudential policy rules.

In their model, firms' physical capital can only be funded by banks. Banks do not have enough net worth and have to borrow from households by issuing deposits and bank outside equity. The model further assumes that bankers have the potential to divert a fraction of the bank assets. The fraction of the assets that banks can divert increase with the ratio of the bank outside equity to assets. Because of this potential moral hazard, bankers face a twofold borrowing constraint. First, there is a limit to the amount of assets that a bank can fund with a given amount of net worth. Second, the borrowing constraint becomes tighter when the bank issues too much outside equity.

[1]For example, Lorenzoni (2008), Bianchi (2011), Korinek (2011) and Stein (2012).

[2]For instance, time-varying capital requirements are analyzed by Kashyap and Stein (2004) and Dewatripont and Tirole (2012). Admati et al. (2010) and Hanson et al. (2011) suggest that banks maintain a capital ratio that is substantially higher than that determined by the market. See also other recent papers, e.g., Perotti and Suarez (2009), Elliott (2011), Borio and Zhu (2012), and Galati and Moessner (2012).

In the GKQ framework, banks prefer outside equity financing to deposits financing. Outside equity has a hedging effect for the bank because its cash-outflows are matched with the cash-inflows from the bank assets. When a financial shock leads to a deterioration of the bank assets, outside equity can absorb part of the bank's loss, so that it has a stabilizing effect on asset prices and production in the economy. However, the bank does not fully internalize the stabilizing effect of outside equity. As a result, the bank chooses a fraction of outside equity which is lower than the socially desirable level.

GKQ show that this problem can be overcome by a macroprudential policy which subsidizes and promotes the issuance of the bank outside equity. The policy helps banks internalize the hedging effect of outside equity. Another implication of such a policy is that the bank capital ratio is high in normal times and low in bad times, so the policy has an effect that is similar to a 'countercyclical capital requirement'. Also, such a policy has the side benefit that it enhances the equilibrium value of the bank net worth, partially relaxing the borrowing constraint. Thus, the bank can fund more bank assets than that under the no-policy regime.

This paper proposes an alternative macroprudential policy in the GKQ framework. The alternative policy differs from that of GKQ in the way the equity subsidy rate is determined. In the GKQ rule, the subsidy responds inversely to the shadow cost of the deposit issuance to indirectly affect the outside equity-to-asset ratio. In our rule, the subsidy responds directly to the aggregate level of the outside equity-to-asset ratio. It has two advantages: first, the equity-to-asset ratio is a more practical measure because it is easier to observe than the shadow cost of the deposit; second, as we demonstrate, the subsidy responding to the aggregate equity-to-asset ratio achieves higher welfare.

The alternative rule improves welfare for two reasons. First, it has a better stabilization effect on bank assets after the economy is hit by a financial shock. It is because the subsidy rate reacts to shocks more strongly than the GKQ rule after the shock, reducing the outside equity subsidy more. The lower subsidy results in a lower outside equity ratio, encouraging more deposit issuance in bad times. In other words, our rule is more in line with the countercyclical capital requirements in Hanson et al. (2011). It has the following implications: as in the GKQ rule, banks should keep a high equity-to-asset ratio in normal times to hedge the financial risk; and unlike the GKQ rule, the central bank should allow for more deposit issuance in bad times to stabilize the economy. The quantitative effect of the alternative rule in bad times is consistent with the macroprudential capital policy tool of the Bank of England (Harimohan and Nelson (2012)).

Second, our policy rule has the side benefit that the bank assets and production in normal times are slightly higher than under the GKQ rule. It means that the better stabilization effect is not at the cost of production and consumption in normal times. Indeed, the level of outside equity subsidy under our rule is similar to that of the GKQ rule. But our rule achieves a higher level of bank assets in the pre-crisis economy because the better post-shock stabilization effect improves the bank's condition in normal times. The results show that our policy raises the bank's private value of the net worth to a higher level. It results in a higher leverage and hence, a higher level of total bank assets. Our analysis sheds light on the indirect effect of the macroprudential policy on security prices and demonstrates that the stabilization of policy has a significant *positive* effect on the pre-crisis economy.

The rest of the paper is structured as follows. Section 1.2 describes the model and the macroprudential policy. Section 1.3 shows simulation results and analyzes the performances of different policy rules. Section 1.4 concludes the paper.

1.2 The Model

The framework is from GKQ. It is a DSGE model with financial frictions. There are four sectors in the model: households, goods producers, capital producers, and banks. The summary of the system of equations is shown in Appendix 1.5.1.

1.2.1 Resource Constraints

The resource constraint of the economy is

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{1.1}$$

where Y_t is domestic final output, C_t is consumption, and I_t is investment. Here $f(\cdot)$ is an adjustment cost function of investment with $f(1) = f'(1) = 0$ and $f''(x) > 0$, $\forall x > 0$.[3]

Following GKQ, the financial crisis is modelled as a negative exogenous shock to the aggregate physical capital stock:

$$K_{t+1} = \psi_{t+1} S_t, \tag{1.2}$$

$$S_t = (1 - \delta) K_t + I_t, \tag{1.3}$$

where K_{t+1} is the aggregate capital stock at the beginning of time $t+1$, S_t is the accumulated aggregate capital at the end of time t, and ψ_{t+1} is the capital quality shock from time t to $t+1$. The shock ψ_{t+1} (> 0) is an i.i.d. process with unconditional mean 1. Eq. (1.3) is the capital accumulation function and δ is the depreciation rate.

1.2.2 Households

A representative household has a continuum of members with mass 1, where ζ fraction of the members are workers and $1 - \zeta$ fraction are bankers. Workers supply labor to firms to gain wages. Bankers work as bank managers, make decisions on the bank balance sheet and get paid by bank dividends when they are forced to exit the banking sector. In each period, bankers have probability σ of exiting the banking sector to become workers, and workers become new bankers with probability $\frac{(1-\sigma)(1-\zeta)}{\zeta}$. In this way, the ratio of bankers to workers remains constant over time.

[3] In the simulation, we follow GKQ and specify f as a quadratic function: $f(x) = \Psi (x-1)^2$.

The expected discounted lifetime utility of the household takes the form[4]

$$\mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, C_{\tau-1}, L_\tau\right) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \frac{1}{1-\gamma} \left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi} L_\tau^{1+\varphi}\right)^{1-\gamma}, \tag{1.4}$$

where $\mathbb{E}_t(\cdot)$ denotes the expectation conditional on the information set at time t, L_t is the labor supply, β is the time discount factor, γ is risk aversion, h is the habit parameter, χ is the weight parameter of labor, and φ is the inverse Frisch labor elasticity.

It is assumed that households do not lend any funds directly to firms. They can only lend to banks. Households can choose to buy either risk-free debt (deposits, D_t) or bank outside equity (e_t). The household maximizes her expected utility by choosing consumption, labor supply, deposits and outside equity subject to the budget constraint:

$$C_t + D_t + q_t e_t = W_t L_t + \Pi_t + R_t D_{t-1} + R_{e,t} q_{t-1} e_{t-1}, \tag{1.5}$$

where W_t is the wage rate, Π_t is the profit transfer from capital producers and banks, q_t is the price of the bank equity, R_t is the gross risk-free return on deposits, and $R_{e,t}$ is the gross return on bank equity from time $t-1$ to t. Let $U_{C,t}$ denote the marginal utility to consume and $\Lambda_{t,\tau}$ the stochastic discount factor from time t to τ:

$$U_{C,t} \equiv \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right)^{-\gamma} - \beta h \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi} L_{t+1}^{1+\varphi}\right)^{-\gamma}, \tag{1.6}$$

$$\Lambda_{t,\tau} \equiv \beta^{\tau-t} \frac{U_{C,\tau}}{U_{C,t}}. \tag{1.7}$$

The optimality condition implies the Euler equations and the labor supply function:

$$R_{t+1} \mathbb{E}_t \left(\Lambda_{t,t+1}\right) = 1, \tag{1.8}$$

$$\mathbb{E}_t \left(\Lambda_{t,t+1} R_{e,t+1}\right) = 1, \tag{1.9}$$

[4]The utility function follows Guvenen (2009) and Greenwood, Hercowitz and Huffman (1988). First, there is no wealth effect on labor supply. Second, it produces labor volatility with little cost of complexity. Third, habit formation improves the quantitative performance of the model.

$$\mathbb{E}_t U_{Ct} W_t = \chi \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi-\gamma} \right) L_t^{\varphi}. \tag{1.10}$$

1.2.3 Goods Producers

Competitive goods producers (henceforth 'firms') accumulate physical capital and produce identical final output using capital and labor. It is assumed that firms lack funds and hence, have to borrow from banks by issuing firm equity. They borrow without any friction by committing to pay the entire profit to banks. The production function is

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{1.11}$$

where A_t denotes aggregate productivity. For simplicity, it is assumed that $A_t = A = 1$.[5] The first order condition with respect to labor is

$$W_t = (1-\alpha) \frac{Y_t}{L_t}. \tag{1.12}$$

Let Z_t denote the gross profit per unit of capital held by the firm:

$$Z_t = \frac{Y_t - W_t L_t}{K_t} = \alpha \frac{Y_t}{K_t} = \alpha A \left(\frac{L_t}{K_t} \right)^{1-\alpha}. \tag{1.13}$$

Since all profits go to banks via firm equity, firms have no profit left in each period. The return on the firm equity is

$$R_{k,t} = \frac{[Z_t + (1-\delta) Q_t] \psi_t}{Q_{t-1}}, \tag{1.14}$$

where Q_t is the price of capital. We normalize the firm equity so that it corresponds to one unit of capital. Under such normalization, Z_t is equal to the profit per unit of firm equity and Q_t is also the price of the firm equity. Similarly, the bank outside equity is

[5] By assuming this, GKQ focus on the impact of the capital quality shock (i.e., 'crisis') and stabilization of the financial system. We follow GKQ and abstract from normal business cycle shock.

entitled to one unit of the bank asset, so the return on the bank equity is

$$R_{e,t} = \frac{(Z_t + (1-\delta)q_t)\,\psi_t}{q_{t-1}}. \tag{1.15}$$

1.2.4 Capital Producers

Capital producers produce capital with flow-variable adjustment costs. They transfer their profit back to households in each period because households have the ownership. Given the capital price Q_t, capital producers choose investment I_t to maximize their profit:

$$\max \mathbb{E}_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau} \left\{ Q_\tau I_\tau - I_\tau \left[1 + f(\frac{I_\tau}{I_{\tau-1}}) \right] \right\}.$$

In the optimum, the marginal cost of capital production equals the price of capital:

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t \left[\Lambda_{t,t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 f'(\frac{I_{t+1}}{I_t}) \right]. \tag{1.16}$$

1.2.5 Banks

In this section, we introduce the banking sector and the central bank macroprudential policy. Banks issue outside equity (e_t) to households and take deposits (d_t) from them, lend money to firms, and keep profits as net worth (n_t).[6] As in GKQ, the central bank implements a macroprudential policy which taxes the bank assets and subsidizes the bank outside equity. With such macroprudential policy, the bank balance sheet condition is

$$(1+\tau_t^k)Q_t s_t = n_t + (1+\tau_t^e)q_t e_t + d_t, \tag{1.17}$$

where s_t is the firm equity that banks buy from goods producers, τ_t^k is the tax on each unit of bank asset, and τ_t^e is the subsidy for bank outside equity. The baseline model (i.e., 'no-policy regime') is a special case that both τ_t^k and τ_t^e are zero.

[6] In GKQ, the following two assumptions ensure that banks do not have sufficiently high net worth to avoid borrowing from households. First, new bankers do not hold sufficient net worth when they enter the banking sector. Second, banks have a constant probability of exiting the banking sector in every period.

The GKQ policy and our alternative policy differ in the way in which the subsidy level τ_t^e is chosen. Once τ_t^e is chosen, the tax level τ_t^k is determined by the fiscal neutrality of the macroprudential policy,

$$\tau_t^k = X_t \tau_t^e, \tag{1.18}$$

where X_t is the aggregate counterpart of the individual level of the outside equity-to-asset ratio,[7]

$$x_t = \frac{q_t e_t}{Q_t s_t}. \tag{1.19}$$

Define the bank leverage ratio ϕ_t as the ratio of bank assets to net worth,

$$\phi_t = \frac{Q_t s_t}{n_t}. \tag{1.20}$$

A law of motion of an individual bank's net worth is

$$n_t = R_{k,t} Q_{t-1} s_{t-1} - R_t d_{t-1} - R_{e,t} q_{t-1} e_{t-1}. \tag{1.21}$$

The objective of a banker is to maximize his expected discounted value of the bank dividend that he will have when he is forced to exit the banking sector:

$$V_t = \mathbb{E}_t \left[\sum_{\tau=t+1}^{\infty} (1-\sigma)\sigma^{\tau-t-1} \Lambda_{t,\tau} n_\tau \right]. \tag{1.22}$$

A moral hazard problem is embedded between shareholders (households) and bankers as in Gertler and Karadi (2011). After the banker has obtained funds from households, he can transfer a fraction of the bank assets back to his family. The fraction of diversion from bank assets (Θ_t) is assumed to depend on the current composition of the bank's borrowing, x_t:

$$\Theta(x_t) = \theta \left(1 + \varepsilon x_t + \frac{\kappa}{2} x_t^2\right), \tag{1.23}$$

[7] We use lower case letters for individual variables (x_t,d_t,s_t,e_t,n_t) and capital letters for aggregate variables (X_t,D_t,S_t,E_t,N_t).

where $\theta > 0$, $\varepsilon < 0$, and $\kappa > 0$.[8] If the banker diverts a fraction of the bank assets, the bank defaults. Being aware of the banker's behavior, households restrict the amount of their lending to ensure that the banker's payoff from diverting funds does not exceed his value of staying in the banking sector. Therefore, the banker will not choose to divert funds in equilibrium, but he faces a borrowing constraint:

$$V_t \geq \Theta(x_t) Q_t s_t. \tag{1.24}$$

The detailed derivation of the banker's optimal decision in the absence of policy is shown in Appendix 1.5.3. The problem in the presence of macroprudential policy can be solved analogously. Under the macroprudential policy of (1.17), the closed form of V_t is

$$V_t(s_t, x_t, n_t) = \left[\left(\mu_{s,t} - \tau_t^k \nu_t\right) + \left(\mu_{e,t} + \tau_t^e \nu_t\right) x_t\right] Q_t s_t + \nu_t n_t, \tag{1.25}$$

where the Lagrange multipliers (LMs) in (1.25) are recursively defined as

$$\nu_t \equiv \mathbb{E}_t \left(\Lambda_{t,t+1} \Omega_{t+1}\right) R_{t+1}, \tag{1.26}$$

$$\mu_{s,t} \equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{k,t+1} - R_{t+1}\right)\right], \tag{1.27}$$

$$\mu_{e,t} \equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1} - R_{e,t+1}\right)\right], \tag{1.28}$$

$$\Omega_{t+1} \equiv (1 - \sigma) + \sigma \left[\phi_{t+1} \left(\mu_{s,t+1} + x_{t+1} \mu_{e,t+1}\right) + \nu_{t+1}\right]. \tag{1.29}$$

Above, Ω_{t+1} is the shadow price of the net worth tomorrow, ν_t is the bank's private cost of issuing deposits (or, the private value of a unit of bank net worth), and $\mu_{e,t}$ is the private cost of issuing deposits in excess of outside equity. In the steady state $\mu_{e,t}$ is positive. This is because bank outside equity can help banks hedge against the financial

[8] The parameter values of ε and κ in GKQ and in this paper are such that the marginal fraction of diversion $\Theta'(x_t)$ is positive. The intuition is that at the margin, it is more difficult to divert the bank assets funded by short-term debt than those funded by outside equity. Since the bank equity is contingent liability, it is more difficult to monitor by outside equity holders. This idea comes from Calomiris and Kahn (1991).

shock and consequently, the cost of outside equity is lower than that of deposits from the perspective of the bank. The expression $\mu_{s,t} + x_t\mu_{e,t}$ is the net profit of bank assets with the balance sheet structure x_t. As the borrowing constraint (1.24) is binding, by (1.20) and (1.25), the optimal bank leverage ratio is

$$\phi_t = \frac{\nu_t}{\Theta(x_t) - (\mu_{s,t} + x_t\mu_{e,t})}. \tag{1.30}$$

The macroprudential policy rule in GKQ chooses the bank equity subsidy depending on the shadow cost of the bank deposit, ν_t:

$$\tau_t^e = \frac{\tau_1}{\nu_t}, \tag{1.31}$$

where τ_1 is a policy parameter. In our alternative rule, called *capital ratio rule*, the central bank chooses the bank equity subsidy based on the *aggregate* bank outside equity ratio, X_t:[9]

$$\tau_t^e = \tau_0 X_t + A_{\tau 0}, \tag{1.32}$$

where $A_{\tau 0}$ is the level parameter and τ_0 is the slope parameter, and they can be negative. When τ_0 is positive, the central bank offers a progressive subsidy.

If we consider the evolution of aggregate net worth in the banking sector, the dynamics of new bankers and old bankers needs to be taken into account. It is assumed that the initial wealth of new bankers $N_{y,t}$ is a fixed fraction ξ of the bank assets at the end of the period t,

$$N_t = \sigma\left(R_{k,t}Q_{t-1}S_{t-1} - R_t D_{t-1} - R_{e,t}q_{t-1}E_{t-1}\right) + N_{y,t}, \tag{1.33}$$

$$N_{y,t} = \xi R_{k,t}Q_{t-1}S_{t-1}.$$

[9] The idea is from a number of studies in the literature such as Perotti and Suarez (2009) and Hanson et al. (2011). They suggest that macroprudential policy should focus on regulations of bank capital and capital requirements. Here, X_t is the aggregate level of the outside equity ratio so that one individual bank cannot affect the subsidy level τ_t^e. In equilibrium, all individual banks choose the same level of x_t.

Following GKQ, welfare is measured by the household's lifetime utility (1.4) at the steady state (i.e., conditional on being in the pre-crisis economy). We compare welfare under different policy rules using consumption equivalent. More details of the welfare description are shown in Appendix 1.5.6.

1.2.6 Frictions in the Model

In the GKQ model, the source of financial frictions is the moral hazard problem. One outcome of the moral hazard problem is that the total bank assets that banks can fund are limited by the borrowing constraint. Banks do not have sufficient net worth, so they have to borrow from households. Due to the banker's ability to divert funds, households restrict the amount of funds they provide to the banker. Therefore, with a given amount of net worth, there is a limit to the assets that the bank can fund.

Given the limit on bank borrowing, there is a trade-off between issuing bank outside equity and issuing deposits. On one hand, from the perspective of the bank, the private cost of issuing deposits is higher than that of outside equity because bank outside equity can help banks hedge against the financial risk. On the other hand, a higher ratio of outside equity on the balance sheet incentivizes the banker to divert more funds, tightening the borrowing constraint. As a result, the bank's optimal choice is a mixture of outside equity and deposits.

In the following analysis, we describe how the macroprudential policy (1.17)-(1.18) alters the outside equity ratio, and thereby affects the moral hazard problem. These qualitative features are common to the GKQ policy rule and the capital ratio rule. We start with the direct effect of the policy on the bank's balance sheet.

Effect 1 *A higher level of outside equity subsidy τ_t^e raises the outside equity ratio x_t in the steady state and tightens the borrowing constraint.*

From the perspective of the bank, a higher level of outside equity subsidy τ_t^e gives extra revenue from issuing bank outside equity, so it increases the outside equity ratio x_t in the steady state. A higher outside equity ratio raises the banker's fraction of diversion, and according to (1.30), it makes the borrowing constraint tighter. We call this negative effect of the policy on the bank's borrowing constraint *moral hazard effect*.

On the other hand, the macroprudential policy has a general equilibrium effect on the security prices and thereby indirectly affects the borrowing constraint.

Effect 2 *A higher level of subsidy reduces the bank's extra cost of deposits in excess of outside equity μ_e and raises the net profit of bank assets μ_s, both of which make the moral hazard cost less severe and relax the bank's borrowing constraint.*

The macroprudential policy changes the bank's private cost/value of outside equity and net worth in equilibrium. We call this positive, general equilibrium effect of the policy *cost reduction effect*. The details of this effect are the following.

As the central bank raises the outside equity subsidy, banks supply more outside equity and less deposits, so the expected return on outside equity $R_{e,t+1}$ goes up and the return on deposits R_{t+1} goes down. From Eq. (1.28), it lowers $\mu_{e,t}$ in the steady state; and from Eq. (1.27), a lower R_{t+1} raises the profit per unit of bank asset, and hence $\mu_{s,t}$ increases in the steady state. Quantitatively, the overall effect of the subsidy on the net profit of bank asset $\mu_{s,t} + x_t\mu_{e,t}$ turns out to be positive.[10] From Eq. (1.29), a higher net profit of bank asset has an amplification effect on the shadow price of the net worth Ω_t because $\mu_{s,t} + x_t\mu_{e,t}$ is leveraged by ϕ_t. As a result, a higher bank equity subsidy τ_t^e induces a higher level of Ω_t in the steady state. The higher level of Ω_t means that banks value the future net worth more, and from (1.26)-(1.28), it has a reinforcing effect on LMs in the steady state: ν_t is increased, $\mu_{s,t}$ is further increased, and the initial drop of $\mu_{e,t}$ is partially cancelled.

In summary, there are two channels through which the outside equity subsidy affects the leverage ratio: Effect 1 shows that a higher level of subsidy raises the outside

[10] In the steady state, the return on the firm equity R_k is higher than both the costs of issuing deposits R_e and bank equity R, so the credit spread $R_k - R$ is larger than the cost difference $R_e - R$. Moreover, the outside equity ratio x is smaller than 1. From (1.27)-(1.28), μ_s is much larger than $x\mu_e$ in the steady state.

equity ratio and tightens the borrowing constraint, while Effect 2 demonstrates that the subsidy relaxes the borrowing constraint by changing security prices. When the cost reduction effect on the bank leverage ratio dominates the moral hazard effect, banks benefit more from the hedging effect of outside equity. In that sense, the financial friction in the banking sector is reduced and therefore, banks can raise more assets in equilibrium. From this aspect, the central bank can choose a subsidy level that achieves the highest bank assets in the pre-shock economy.

1.3 Simulation Results

In this section, we compare the simulation results of different macroprudential policy rules to show how they mitigate the negative impact of the financial shock and improve welfare. Section 1.3.1 shows the simulation results. Section 1.3.2 provides the impulse responses of variables after the financial shock and explains why the capital ratio rule has a better stabilization effect on the bank assets and production than the GKQ rule. Finally, Section 1.3.3 investigates how the better stabilization effect further results in a higher steady state level of bank assets and consumption.

Following GKQ, we conduct the first-order approximation around the 'risk-adjusted steady state' (i.e., the 'risky steady state' in Coeurdacier, Rey and Winant (2011)) instead of around the deterministic steady state, so that agents' perceptions of the possible future risk have an impact on the steady state values. The risky steady state refers to "the point where agents choose to stay at a given date if they expect future risk and if the realization of shocks is 0 at this date" [Coeurdacier et al. (2011)], and in this paper, it represents the pre-crisis economy or the economy in normal times. Henceforth, we refer to the risky steady state simply as steady state. To compute the risky steady state, we use the iteration method that follows Coeurdacier et al. (2011) and GKQ, which we explain in detail in Appendix 1.5.5. Also, the welfare computation is described in Appendix 1.5.6.

There are 16 parameters in the baseline model. The parameter values are from

GKQ and they are summarized in Table 1.1.

Table 1.1: Parameters

β	0.99	discount factor
γ	2	relative risk aversion
h	0.75	habit formation coefficient
χ	0.25	weight coefficient of labor
φ	0.33	inverse Frisch elasticity of labor supply
α	0.33	share of capital
δ	0.0025	physical capital depreciation rate
Ψ	1	elasticity of the price of capital to investment
σ	0.9685	probability of bank survival
ξ	0.00289	households transfer ratio to new banks
θ	0.264	moral hazard parameter
ε	-1.21	bank's diversion parameter (linear term)
κ	13.41	bank's diversion parameter (quadratic term)
$E(\psi_t)$	1	mean of the financial shock
$std(\psi_t)$	0.69%	standard deviation of the financial shock

1.3.1 The Risky Steady State

The risky steady state values of key variables are shown in Table 1.2. For each policy scenario, the results are under the optimal policy parameter(s). The first column is levels and the second column is standard deviations.

Table 1.2 shows that in comparison with the no-policy regime, the GKQ rule achieves 0.53 percent consumption equivalent. But the capital ratio rule improves welfare to a higher level of 1.62 percent consumption equivalent. That is, under the capital ratio rule, the financial frictions in the model are more mitigated. The results also show that the frictionless economy has the consumption equivalent of 4.8 percent and hence, the capital ratio rule recovers about one third of the welfare loss.[11]

Figure 1.1 shows the welfare surface under the capital ratio rule. In Figure 1.1, the Y-axis is the steady state level of τ^e instead of τ_0 for the exposition purpose.[12] The

[11] We can derive the equilibrium of a frictionless economy by solving the social planner's problem, choosing $(Y_t, L_t, C_t, I_t, S_t)$ to maximize the household's utility subject to the resource constraints (3.30)-(1.3) and (1.11).

[12] There is a one-to-one mapping between the steady state level of τ^e and $\tau_0 > 0$ for a given value of $A_{\tau 0}$ because X_t increases with τ_0.

Table 1.2: The Risk Steady States and Standard Deviations[13]

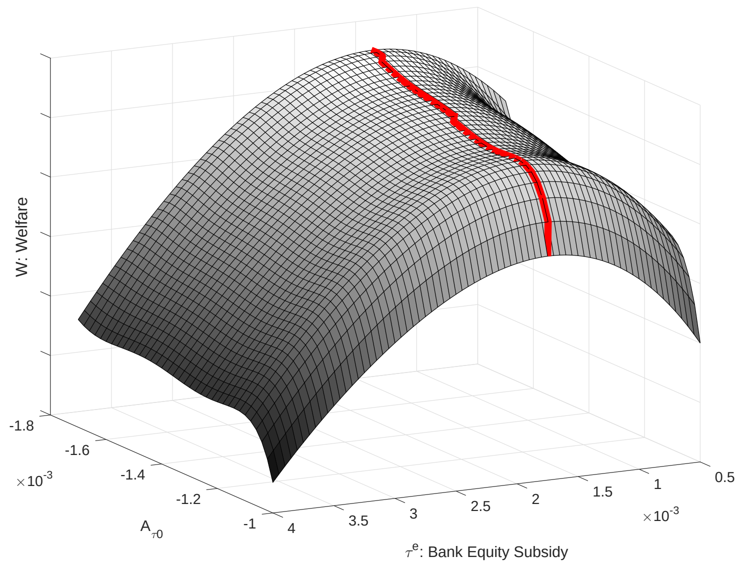
		No policy		GKQ rule		Capital ratio rule	
		Steady state	Std. Dev.	Steady state	Std. Dev.	Steady state	Std. Dev.
Output	Y	23.58	0.9526	24.17	0.9573	24.37	0.9334
Consumption	C	18.42	0.7165	18.82	0.7147	18.95	0.6974
Labor	L	8.10	0.2414	8.25	0.2390	8.30	0.2266
Capital	K	206.29	12.7281	214.25	12.9550	216.91	12.4599
Net worth	N	32.48	4.9305	31.84	6.1674	31.33	7.4127
Risk-free return	$R(\%)$	1.02	0.0017	0.985	0.0021	0.974	0.0022
Risky return	$R_k(\%)$	1.27	0.0141	1.227	0.0108	1.211	0.0102
Credit spread	$R_k - R(\%)$	0.25	0.0155	0.24	0.0125	0.237	0.0119
Outside equity ratio	x	0.1036	0.0372	0.1619	0.0489	0.1556	0.1130
Leverage ratio	ϕ	6.4876	0.7195	6.7123	0.9813	6.9094	1.2748
Deposit cost	ν	1.6051	0.1284	1.7152	0.1552	1.7546	0.1377
Excess equity cost	μ_e	0.047	0.0018	0.030	0.0042	0.017	0.0054
Bank asset profit	μ_s	0.2447	0.6862	0.3109	0.1981	0.3187	0.0827
Overall profit	$\mu_s + x\mu_e$	0.2496	0.6843	0.3157	0.1960	0.3213	0.0799
Subsidy	τ^e	0	N/A	0.0016	1.45×10^{-4}	0.00155	2.36×10^{-4}
Tax	τ^k	0	N/A	2.6×10^{-4}	1.02×10^{-4}	2.4×10^{-4}	5.39×10^{-4}
Consumption equivalent	$\Gamma(\%)$	0	N/A	0.5264	N/A	1.6184	N/A
Financial shock	ψ	1	0.69%	1	0.69%	1	0.69%

red line on the surface shows the optimal welfare point for each value of $A_{\tau 0}$, and the optimal point is found at $(\tau_0^*, A_{\tau 0}^*) = (0.0209, -0.0017)$. Whereas the optimum is found on the boundary of the set of parameter values that guarantee the Blanchard-Kahn condition, the capital ratio rule performs better than the GKQ rule for a wide range of parameter values: $\tau_0 \in (0.0045, 0.0209)$ and $A_{\tau 0} \in (-0.0017, 0)$.

The results in Table 1.2 show that the GKQ rule and our rule are similar regarding the optimal level of bank equity subsidy τ^e (0.0016 and 0.00155, respectively) and the outside equity ratio x (16.19% and 15.56%, respectively) in the steady state. However, under the capital ratio rule, the bank's private value of net worth (ν_t) and the leverage ratio are higher than under the GKQ rule, which gives rise to higher bank assets, output, and consumption. Also, under the capital ratio rule, the standard deviations of the real sector variables are lower than under the GKQ rule. In the next section, we demonstrate that the capital ratio rule has a better stabilization effect on bank assets and production in the economy after a financial shock. Then in Section 1.3.3, we explain how the better stabilization effect under our rule, in turn, leads to a higher level of bank assets, output, and consumption in the pre-shock economy.

[13]The results are robust to the choice of parameters, and more details are shown in Appendix 1.5.4.

Figure 1.1: Welfare under the Capital Ratio Rule



Note: The optimal point is when $(\tau_0^*, A_{\tau 0}^*) = (0.0209, -0.0017)$, and it corresponds to $\tau^e = 0.00155$.

1.3.2 The Policy Response to Shock

In this section, we conduct an impulse response analysis and demonstrate that under the capital ratio rule, the equity subsidy reacts more strongly to shocks with the result that the economy has less fluctuations than that under the GKQ rule. Figure 1.2 shows the impulse responses of relevant variables under different rules. The shock is a decrease of ψ_t by one standard deviation. All variables are in log deviations from the steady state except the credit spread, which is presented as the actual deviation.

The response in the no-policy regime is provided as a benchmark. In the no-policy regime, when the shock occurs, the total bank assets deteriorate and banks suffer loss, so the net worth N_t goes down. Since the decrease in the net worth makes the bank borrowing constraint tighter, banks choose to lower the moral hazard cost Θ_t by reducing the outside equity ratio x_t. Consequently, bank outside equity e_t drops by 15% and bank deposits D_t goes up by 5%. The drop of capital K_t leads to an increase in the marginal product of capital and hence, the return on the bank asset $R_{k,t}$ goes up. The impact of the shock on $R_{k,t}$ is larger than that on the risk-free return R_t, so the credit spread $R_{k,t} - R_t$ increases. The increase in the credit spread raises the net profit of the

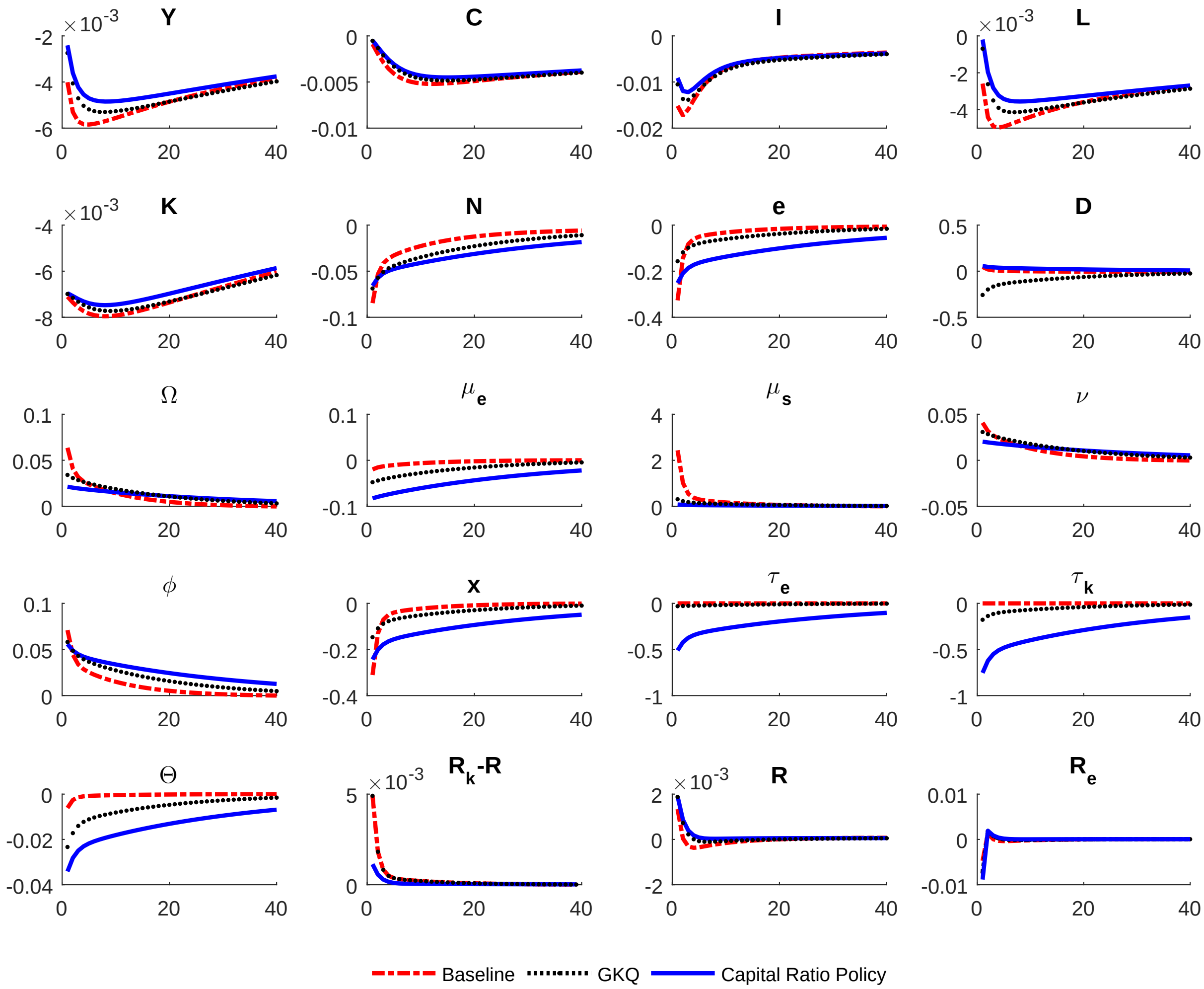


Figure 1.2: Baseline Model, GKQ Policy and Capital Ratio Policy

bank asset $\mu_{s,t} + x_t\mu_{e,t}$. From Eq. (1.30), lower Θ_t and higher $\mu_{s,t} + x_t\mu_{e,t}$ result in an increase in the leverage ratio ϕ_t.

The positive response of ϕ_t implies that banks can obtain more assets for each unit of net worth, and a higher $\mu_{s,t} + x_t\mu_{e,t}$ indicates that each unit of net worth yields a higher profit. From Eq. (1.29), the higher net profit $\mu_{s,t}+x_t\mu_{e,t}$ is leveraged by ϕ_t, which leads to a positive response of the shadow price of the net worth Ω_{t+1}. From (1.26)-(1.28), a higher Ω_{t+1} raises the LMs ν_t, $\mu_{s,t}$ and $\mu_{e,t}$ in the following period, which helps to maintain the positive response of the leverage ratio. In this way, although the net worth decreases, banks lowering the outside equity ratio helps banks mitigate the negative impact of the financial shock through the long-lasting positive effects on the leverage ratio. This is the self-recovering procedure of the banking sector after a financial shock that is present even when no policy is conducted.

Before looking at the impulse responses under time-varying macroprudential policies, we briefly analyze the effect of *constant subsidy rule* in which the subsidy rate is constant over time (i.e. $\tau_t^e = \tau$).[14] From Effect 1, the subsidy raises the outside equity ratio x_t in the steady state. Because of the hedging effect of outside equity, a high level of x_t helps to absorb part of the bank's loss caused by the financial shock. Therefore, a higher level of outside equity subsidy has a better hedging effect beforehand. After the shock occurs, however, the subsidy hinders the self-recovering procedure of the banking sector through the leverage ratio. While the shock induces the outside equity ratio to decrease, a high level of subsidy maintains the outside equity ratio at a high level. It raises the moral hazard cost, lowers the leverage ratio and thereby, partially impedes the deposit issuance after the shock. Overall, if the central bank chooses an optimal level of subsidy ($\tau_t^e = \tau = 0.0016, \forall t$), the pre-shock hedging effect dominates the post-shock impeding effect, so the welfare is improved.

Now we look at impulse responses under the GKQ rule. The pre-shock hedging effect of the subsidy is nearly the same as that under the constant subsidy rule because the steady state level of subsidy is very close. However, as the subsidy responds negatively to ν_t under the GKQ rule, the central bank reduces the subsidy by 2% and keeps it low for a long period of time, which causes a 10% decrease in the outside equity ratio x_t. Compared to the constant subsidy rule, the GKQ rule reduces the subsidy after the shock and therefore, the bank's self-recovery procedure is less hindered. As a result, the GKQ rule leads to a longer-lasting increase in the leverage ratio, and thereby a better stabilization effect on the bank assets than the constant subsidy rule.

Compared to the GKQ rule, the capital ratio rule has an even larger stabilization effect on the economy. The analysis of the bank's self-curing mechanism under the

[14] The impulse responses under the constant subsidy rule are not shown in Figure 1.2 because it would congest the figure. They are shown in Figure 1.4 in Appendix 1.5.7.

no-policy regime suggests that reducing the subsidy after the shock helps to reduce the moral hazard cost, raises the leverage ratio ϕ_t and makes the positive response of ϕ_t last for a longer period of time. Being different from the GKQ rule, the capital ratio rule decreases the subsidy by 50% after the shock and keeps it around 20% below the steady state, achieving a lower outside equity ratio. In other words, under the capital ratio rule, the self-recovering procedure through leverage is even less hindered than under the GKQ rule. As a result, it has a better performance in mitigating the consumption and output drop, which is consistent with the smaller second moments of variables (e.g., K_t and C_t) in Table 1.2.

In the context of the capital requirement for bank, the GKQ rule and the capital ratio rule require a similar level in normal times but the latter is more 'countercyclical': it lowers the capital requirement to a larger extent once the shock occurs (see Figure 1.2). A high capital ratio before the shock helps the bank hedge the financial risk, but a lower capital ratio after the shock can reduce the decrease in total bank assets. Our rule is consistent with the literature on a time-varying capital requirements, e.g., Kashyap and Stein (2004) and Hanson et al. (2011). Moreover, the capital ratio rule is in line with the macroprudential capital policy tool of the Bank of England in Harimohan and Nelson (2012).

The above analysis implies that if a macroprudential policy rule encourages bank leverage after a shock, it can better stabilize the economy. In this context, while the GKQ rule has "the flavor of a countercyclical capital requirement" (GKQ, page $S31$), it is quantitatively not so countercyclical. The point is that while the GKQ model is rich enough to decompose the permanent, "prudential" component of a policy (i.e., discouraging banks from too much leverage on average) and the "countercyclical" component of a policy (i.e., encouraging banks to invest in assets in bad times), they do not fully distinguish these two components when proposing their policy rule. The capital ratio rule we propose is the most parsimonious policy rule that can affect both the prudential and the countercyclical component separately with an easily-observed target.

Besides, there is one advantage of choosing the outside equity ratio as a policy target that the response of x_t to shocks is more persistent, achieving a more countercyclical capital requirement. This is because x_t is very sensitive to the response of the subsidy to shocks. When the shock occurs, x_t goes down and under the capital ratio rule, a lower level of x_t reduces the subsidy τ_t^e, which in turn maintains x_t at a lower level. It results in a much more persistent response of x_t to shocks. In contrast, the response of ν_t to shocks is relatively small when the central bank applies different rules. For this reason, the policy rule in which the subsidy responds to the outside equity ratio x_t with only one parameter, that is, $A_{\tau 0} = 0$ in (1.32), still outperforms the GKQ rule.

For these reasons, if we modify the shadow value-targeting policy rule of GKQ as

$$\tau_t^e = \tau_A + \tau_B \nu_t + \tau_C \mu_{e,t}, \text{[15]} \tag{1.34}$$

it would perform better partially because it allows separating the prudential part and the countercyclical part and because $\mu_{e,t}$ is a target that is persistent though not easily observed. In the next section, we demonstrate that the better stabilization of the post-shock economy, in turn, leads to higher investment in the pre-shock economy and that it is mainly because the policy reduces the bank's financing cost.

1.3.3 The Second-moment Effect of Different Rules

On one hand, the better stabilization of the capital ratio rule in the post-shock economy explained in Section 1.3.2 positively contributes to the welfare of the risk-averse household. On the other hand, Table 1.2 shows that while the level of the equity subsidy is similar across different rule, the level of bank leverage, physical capital and output in the steady state is higher under the capital ratio rule, which also contributes to the higher welfare. In this section, we attempt to detect the mechanism through which the

[15]This policy rule is optimized when the parameter values are $\tau_A = 0.22$, $\tau_B = -0.128$ and $\tau_C = -16.6$. I acknowledge the referee for suggesting this policy rule.

better stabilization under the capital ratio rule results in a higher level of bank assets, and therefore higher output and consumption in the pre-shock economy.

For this purpose, we will show the results for the GKQ rule, the capital ratio rule, and the constant subsidy rule. We add the constant subsidy rule for comparison because under such a policy rule, we can exclude the second-moment effect of the subsidy's response to shocks on the steady state values. For better comparison, we fix the steady state level of bank equity subsidy across different rules.

We do experiments at various levels of subsidy, and find the result that is common across the policy rules: as we increase the steady state level of the bank equity subsidy τ_t^e, the welfare $\mathcal{W}$ first goes up and then goes down at some point. We find that the highest welfare corresponds to the lowest level of return on physical capital R_k at the steady state. Intuitively, according to the diminishing marginal productivity of capital in the production function, the lowest level of return on capital corresponds to the highest level of capital stock in the steady state, which leads to the highest output and consumption level. So we use R_k as the proxy to see the effect of the policy on welfare. By solving Eqs. (1.26)-(1.30) in the risky steady state system, we can obtain the following proposition that tells us how the return on capital is affected by a policy.

Proposition 1 *In the macroprudential policy framework (1.17), the steady state level of the return on bank assets ($R_{k,t}$) can be expressed as a function of bank outside equity ratio (x_t), subsidy (τ_t^e) and the second moments of variables in the steady state, namely,*

$$R_k = \underbrace{\frac{M_{\mu e} + \tau^e M_\nu}{M_{\mu s}} \frac{\Theta(x)}{\Theta'(x)}}_{S_1} + \underbrace{\frac{\nu - \mu_e x}{\nu + \mu_s}}_{S_2}. \tag{1.35}$$

Here, $M_{\mu e}$, M_{ν} and $M_{\mu s}$ are second moments of μ_e, ν and μ_s, respectively, and they are

$$M_{\nu} = 1 + cov(\widehat{\Omega}, \widehat{\Lambda}), \tag{1.36}$$

$$M_{\mu e} = -\Lambda \left[(R_e - R) + (R_e - R)cov(\widehat{\Lambda}, \widehat{\Omega}) + R_e cov(\widehat{\Lambda}, \widehat{R}_e) + R_e cov(\widehat{\Omega}, \widehat{R}_e) \right], \tag{1.37}$$

$$M_{\mu s} = \Lambda \left(1 + cov(\widehat{\Lambda}, \widehat{\Omega}) + cov(\widehat{\Lambda}, \widehat{R}_k) + cov(\widehat{\Omega}, \widehat{R}_k) \right), \tag{1.38}$$

where $cov(\cdot)$ is the unconditional covariance and '$\wedge$' is the log-deviation from the steady state.

From Eq. (1.35), we can see that the total effect of the policy can be split into the moral hazard effect and the cost reduction effect, which we discussed in Section 1.2.6. The first term S_1 is the moral hazard effect on welfare: $\frac{\Theta(x)}{\Theta'(x)}$ is the inverse of the marginal contribution ratio of the outside equity ratio x on the moral hazard cost, and it increases with the equity subsidy τ^e. As the outside equity ratio x increases with τ^e, the bank's borrowing constraint becomes tighter and hence, the moral hazard effect is higher.

The second term S_2 is the cost reduction effect: it reveals the general equilibrium effect on the private cost/value of outside equity, deposits, and net worth. Recalling (1.26)-(1.28), μ_e is the private cost of the deposit in excess of the outside equity and $\nu + \mu_s$ is the return on the bank asset. When the central bank raises the subsidy level, a higher outside equity ratio x reduces the bank's cost of raising one unit of asset because μ_e is positive in equilibrium. In addition, the general equilibrium effect raises the profit of the net worth μ_s and therefore, S_2 decreases. That is to say, each unit of net worth yields more profit. As a result, the cost reduction effect on welfare is increasing when the subsidy τ^e increases.

Figure 1.3 decomposes the total effect on R_k into the moral hazard effect (S_1) and the cost reduction effect (S_2) at various steady state levels of subsidy, low $S_1 + S_2$ corresponding to low R_k and hence high welfare. In general, as the subsidy increases, the moral hazard effect goes up and the cost reduction effect goes down. Figure 1.3 shows that while the properties of the moral hazard effect under different policy rules

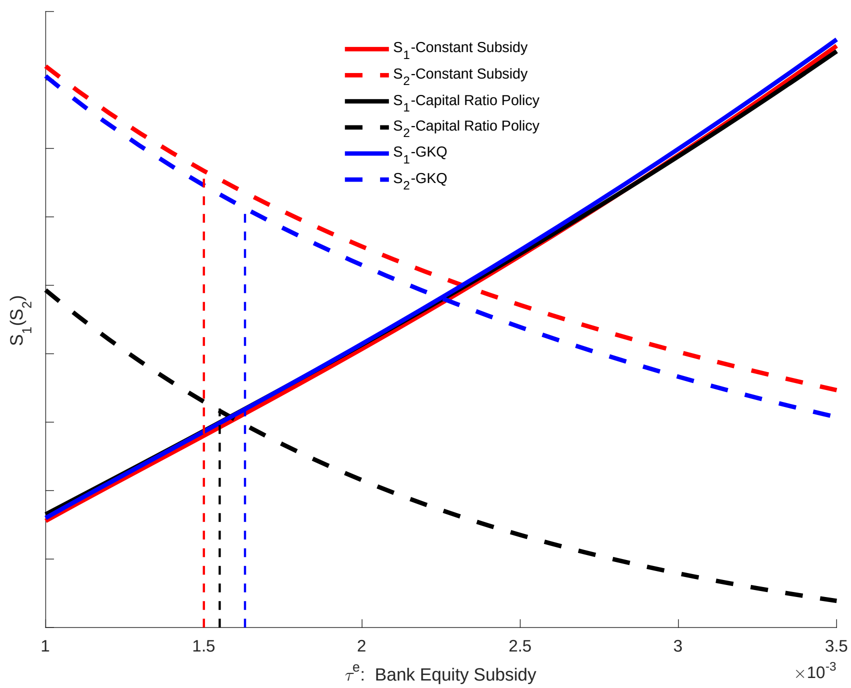


Figure 1.3: Moral Hazard Effect S_1 v.s. Cost Reduction Effect S_2

are nearly the same, the cost reduction effect is significantly lower under the capital ratio rule at any steady state level of subsidy. This is due to the better stabilization of the capital ratio rule. As we discussed in Section 1.3.2, the subsidy reacts more strongly to the financial shock under the capital ratio rule, generating more countercyclical and more persistent Ω_t. In (1.38), because both $R_{k,t}$ and $\Lambda_{t,t+1}$ are countercyclical, the higher countercyclicality of Ω_t raises the covariance terms in $M_{\mu s}$, and it results in a higher μ_s in the risky steady state. As a result, by controlling the level of τ^e, the steady state value of μ_s is higher under the capital ratio rule. It leads to a better cost reduction effect and therefore, a higher steady state level of bank assets.

1.4 Concluding Remark

This paper proposes a welfare-improving macroprudential policy rule in the framework of Gertler, Kiyotaki and Queralto (2012). Our rule chooses the bank equity subsidy rate that responds to the aggregate bank outside equity ratio.

We find that the policy rule proposed in GKQ overlooks the importance of reducing the moral hazard cost after the financial shock. By lowering the bank equity subsidy level after a shock, the moral hazard cost is reduced and banks rely more on the deposit issuance. As a result, bank borrowing is maintained at a higher level, and the bank asset deterioration is more mitigated, achieving a better stabilization effect on asset prices and production. The results show that our rule achieves quantitatively more significant countercyclical capital requirement, which is in line with the macroprudential policy literature.

Our study also sheds light on the mechanism through which the macroprudential policy affects the bank balance sheet through security prices. We find that the response of the subsidy to shocks has a significant, positive impact on the pre-crisis bank asset value. Besides the stabilization effect of the policy itself, this is another important channel through which the policy affects welfare.

1.5 Technical Appendix

1.5.1 The System of Equations under the Alternative Policy

$$Y_t = (\psi_t S_{t-1})^{\alpha} L_t^{1-\alpha}, \tag{1.39}$$

$$Y_t = C_t + \left[1 + f\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{1.40}$$

$$S_t = \psi_t \left[(1-\delta)S_{t-1} + I_{t-1}\right], \tag{1.41}$$

$$R_{t+1}\mathbb{E}_t\left(\Lambda_{t,t+1}\right) = 1, \tag{1.42}$$

$$\mathbb{E}_t\left(\Lambda_{t,t+1}R_{e,t+1}\right) = 1, \tag{1.43}$$

$$j_t = \frac{J_t}{J_{t-1}}, \tag{1.44}$$

$$J_t = C_t - hC_{t-1} - \frac{\chi}{1+\varphi}L_t^{1+\varphi}, \tag{1.45}$$

$$(1-\alpha)\left[1 - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)\right] Y_t = \chi L_t^{1+\varphi}, \tag{1.46}$$

$$\mathbb{E}_t\Lambda_{t,t+1} = \beta \frac{\mathbb{E}_t\left(j_{t+1}^{-\gamma}\right) - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma} j_{t+2}^{-\gamma}\right)}{1 - \beta h \mathbb{E}_t\left(j_{t+1}^{-\gamma}\right)}, \tag{1.47}$$

$$R_{e,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t S_{t-1}}\right)^{1-\alpha} + (1-\delta)q_t\right]\psi_t}{q_{t-1}}, \tag{1.48}$$

$$R_{k,t} = \frac{\left[\alpha\left(\frac{L_t}{\psi_t S_{t-1}}\right)^{1-\alpha} + (1-\delta)Q_t\right]\psi_t}{Q_{t-1}}, \tag{1.49}$$

$$Q_t = 1 + f(\frac{I_t}{I_{t-1}}) + \frac{I_t}{I_{t-1}} f'(\frac{I_t}{I_{t-1}}) - \mathbb{E}_t\left[\Lambda_{t,t+1}\left(\frac{I_{t+1}}{I_t}\right)^2 f'(\frac{I_{t+1}}{I_t})\right], \tag{1.50}$$

$$N_t = \sigma\left[\left(R_{k,t} - x_{t-1}R_{e,t} - R_t + x_{t-1}R_t\right)Q_{t-1}S_{t-1} + R_t N_{t-1}\right] + (1-\sigma)\xi R_{k,t}Q_{t-1}S_{t-1}, \tag{1.51}$$

$$\Theta_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right), \tag{1.52}$$

$$N_t \phi_t = Q_t K_t, \tag{1.53}$$

$$\phi_t = \frac{\nu_t}{\Theta_t - (\mu_{st} + x_t\mu_{et})}, \tag{1.54}$$

$$\nu_t = \mathbb{E}_t(\Lambda_{t,t+1}\Omega_{t+1})R_{t+1}, \tag{1.55}$$

$$\mu_{s,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{k,t+1} - R_{t+1}\right)\right], \tag{1.56}$$

$$\mu_{e,t} = \mathbb{E}_t\left[\Lambda_{t+1}\Omega_{t+1}\left(R_{t+1} - R_{e,t+1}\right)\right], \tag{1.57}$$

$$\Omega_t = (1-\sigma) + \sigma\left[(\mu_{s,t} + x_t\mu_{e,t})\phi_t + \nu_t\right], \tag{1.58}$$

$$\theta\left(\mu_{s,t} + \mu_{e,t}x_t\right)\left(\varepsilon + \kappa x_t\right) = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)\left[\mu_{e,t} + \nu_t\left(\tau_0 x_t + A_{\tau 0}\right)\right], \tag{1.59}$$

$$\left(1 + \tau_t^k\right)Q_tS_t = N_t + \left(1 + \tau_t^e\right)q_te_t + D_t, \tag{1.60}$$

$$x_t = \frac{q_te_t}{Q_tS_t}, \tag{1.61}$$

$$\tau_t^e = \tau_0 X_t + A_{\tau 0}, \tag{1.62}$$

$$\tau_t^k = X_t\tau_t^e. \tag{1.63}$$

$$X_t = x_t. \tag{1.64}$$

1.5.2 The Optimization Problem of Households

Rewrite the budget constraint (1.5) as

$$\Xi_t \equiv C_t + D_{h,t} + q_te_t - W_tL_t - \Pi_t - R_tD_{h,t-1} - \left[Z_t + (1-\delta)q_t\right]\psi_te_{t-1} \leq 0. \tag{1.65}$$

The Lagrangian for the household problem is then

$$\mathcal{L}_1 = \mathbb{E}_t\sum_{\tau=t}^{\infty}\beta^{\tau-t}\left\{\frac{1}{1-\gamma}\left(C_\tau - hC_{\tau-1} - \frac{\chi}{1+\varphi}L_\tau^{1+\varphi}\right)^{1-\gamma} - \lambda_\tau\Xi_\tau\right\}. \tag{1.66}$$

The representative household chooses the variables $(C_t, L_t, D_{h,t}, e_t)$ to maximize (1.4). The first order conditions are

$$\frac{\partial \mathcal{L}}{\partial C_t} = \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right)^{-\gamma} - \lambda_t - \beta h \mathbb{E}_t \left(C_{t+1} - hC_t - \frac{\chi}{1+\varphi} L_{t+1}^{1+\varphi}\right)^{-\gamma} = 0, \tag{1.67}$$

$$\frac{\partial \mathcal{L}}{\partial L_t} = -\chi \left(C_t - hC_{t-1} - \frac{\chi}{1+\varphi} L_t^{1+\varphi}\right)^{-\gamma} L_t^{\varphi} + \lambda_t W_t = 0, \tag{1.68}$$

$$\frac{\partial \mathcal{L}}{\partial D_{h,t}} = \mathbb{E}_t \left(-\lambda_t + \beta R_{t+1} \lambda_{t+1}\right) = 0, \tag{1.69}$$

$$\frac{\partial \mathcal{L}}{\partial e_t} = -\lambda_t q_t + \beta \mathbb{E}_t \left[\lambda_{t+1} \left(Z_{t+1} + (1-\delta) q_{t+1} \psi_{t+1}\right)\right] = 0. \tag{1.70}$$

From the above equations, we can derive (1.8)-(1.10) and (1.15) in Section 1.2.2.

1.5.3 The Optimization Problem of Banks

We use Bellman equations to solve the banker's optimization problem because the value function appears in the moral hazard constraint. Here we demonstrate two different ways to derive the specific form of the value function.

(1) Guess and Verify Method

This method is used in GKQ and Gertler and Kiyotaki (2010). First, we simplify the net worth accumulation function by Eqs. (1.17) and (1.19)

$$n_t = \left[R_{k,t} - R_t(1 - x_{t-1}) - R_{e,t} x_{t-1}\right] Q_{t-1} s_{t-1} + R_t n_{t-1}. \tag{1.71}$$

Here x_t, s_t are the control variables and n_t is the state variable. The Bellman equation is

$$V_{t-1}(s_{t-1}, x_{t-1}, n_{t-1}) = \mathbb{E}_{t-1} \Lambda_{t-1,t} \left\{(1-\sigma) n_t + \sigma \max_{s_t, x_t} \left[V_t(s_t, x_t, n_t)\right]\right\}. \tag{1.72}$$

The guess solution is

$$V_t(s_t, x_t, n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t. \tag{1.73}$$

We need to verify that the solution satisfies the expression (1.72) for all t.

The bank maximizes the objective function subject to the moral hazard constraint, so the Lagrangian is constructed as

$$\mathcal{L}_2 = (1+\lambda_t)[(\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t] - \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_t s_t.$$

The first order conditions with respect to x_t and s_t are

$$(1+\lambda_t) = \frac{\lambda_t\theta\left(\varepsilon + \kappa x_t\right)}{\mu_{e,t}}. \tag{1.74}$$

$$(1+\lambda_t)(\mu_{s,t} + x_t\mu_{e,t}) = \lambda_t\theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right). \tag{1.75}$$

Assuming that the moral hazard constraint is binding, we have

$$V_t(s_t, x_t, n_t) = (\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t = \theta\left(1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2\right)Q_t s_t.$$

Substituting (1.74) into (1.75), we derive banks' optimal outside equity ratio x_t,

$$(\varepsilon + \kappa x_t)\,(\frac{\mu_{s,t}}{\mu_{e,t}} + x_t) = 1 + \varepsilon x_t + \frac{\kappa}{2}x_t^2.$$

Substituting (1.73) into the Bellman Equation (1.72),

$$(\mu_{s,t} + x_t\mu_{e,t})Q_t s_t + v_t n_t = \mathbb{E}_t\Lambda_{t,t+1}\left\{(1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + v_{t+1}\right]\right\}n_{t+1}. \tag{1.76}$$

Denoting $\Omega_{t+1} = (1-\sigma) + \sigma\left[(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1})\phi_{t+1} + \upsilon_{t+1}\right]$, it becomes

$$(\mu_{s,t}+x_t\mu_{e,t})Q_t s_t+\upsilon_t n_t = \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\left(\left[R_{k,t+1} - R_{t+1}(1-x_t) - R_{e,t+1}x_t\right]Q_t s_t + R_{t+1}n_t\right). \tag{1.77}$$

So if we change notations as Eqs. (1.26)-(1.29) in Section 1.2.5, both sides of the expression (1.77) are identical for all t.

$$\begin{aligned} LHS &= \mathbb{E}_t\Lambda_{t,t+1}\Omega_{t+1}\left(\left[R_{k,t+1} - (1-x_t)\,R_{t+1} - x_t R_{e,t+1}\right]Q_t s_t + R_{t+1}n_t\right) \\ &= RHS. \end{aligned}$$

So the Bellman equation is satisfied for any t with the Value function (1.73). This method requires the correct guess of the functional form.

(2) Derivation Method without Guessing the Functional Form

The bank's objective function is (1.22), and the net worth accumulation function (1.21) can be rearranged as

$$g(n_{t+1}, n_t, x_t, s_t) = \Gamma_t n_t - n_{t+1} = 0,$$

where

$$\Gamma_t = \left[R_{k,t+1} - R_{t+1}(1-x_t) - R_{e,t+1}x_t\right]\phi_t + R_{t+1}.$$

So the Lagrangian can be written as

$$\mathcal{L}_3 = \mathbb{E}_t\sum_{\tau=t+1}^{\infty}\sigma^{\tau-t-1}\Lambda_{t,\tau}\left\{(1-\sigma)n_\tau + \Omega_\tau g(n_\tau, n_{\tau-1}, x_{\tau-1}, s_{\tau-1})\right\}.$$

The first order condition with respect to n_{t+1} is

$$\begin{aligned} \Omega_{t+1} &= (1-\sigma) + \sigma\Lambda_{t+1,t+2}\Omega_{t+2}R_{t+2}\dots \\ &+ \sigma\phi_{t+1}\left[\Lambda_{t+1,t+2}\Omega_{t+2}\left(R_{k,t+2} - R_{t+2}\right) + x_{t+1}\Lambda_{t+1,t+2}\Omega_{t+2}\left(R_{t+2} - R_{e,t+2}\right)\right]. \end{aligned}$$

If we use the same notations as Eqs. (1.26)-(1.28), we have

$$\Omega_{t+1} = (1-\sigma) + \sigma\left[\nu_{t+1} + \phi_{t+1}\left(\mu_{s,t+1} + x_{t+1}\mu_{e,t+1}\right)\right]. \tag{1.78}$$

Letting Δ_{t+1} denote the price of the net worth at time $t+1$. Since the objective function (1.22) is linear with respect to the net worth, the bank's value function V_t should be equal to the discounted value of the net worth tomorrow,

$$V_t(n_t) = \mathbb{E}_t \Lambda_{t,t+1} \Delta_{t+1} n_{t+1}. \tag{1.79}$$

We substitute it into the Bellman Equation (1.72) to get the recursive form of Δ_{t+1},

$$\begin{aligned} \mathbb{E}_t \Lambda_{t,t+1} \Delta_{t+1} n_{t+1} &= \mathbb{E}_t\left((1-\sigma)\Lambda_{t,t+1} n_{t+1} + \sigma \mathbb{E}_t \Lambda_{t,t+1}\left[\mathbb{E}_{t+1}\Lambda_{t+1,t+2}\Delta_{t+2} n_{t+2}\right]\right) \\ &= \mathbb{E}_t \Lambda_{t,t+1} n_{t+1}\left[(1-\sigma) + \sigma\left(\Lambda_{t+1,t+2}\Delta_{t+2}\Gamma_{t+1}\right)\right]. \end{aligned} \tag{1.80}$$

The second equality comes from the Law of Iterated Expectation. Since the optimal solution requires that the equality of Eq. (1.80) holds for any t, we can derive the recursive form of Δ_{t+1} by comparing both sides of the equation,

$$\Delta_{t+1} = (1-\sigma) + \sigma \Lambda_{t+1,t+2}\Delta_{t+2}\Gamma_{t+1}. \tag{1.81}$$

Now we show that Ω_{t+1} is the price of net worth at time $t+1$, that is,

$$\Omega_{t+1} = \Delta_{t+1}.$$

Recall that the Lagrangian is

$$\mathcal{L}_3 = \mathbb{E}_t\left[\sum_{\tau=t+1}^{\infty} \sigma^{\tau-t-1}\Lambda_{t,\tau}\left[(1-\sigma)n_\tau + \Omega_\tau\left[n_\tau - \Gamma_{\tau-1}n_{\tau-1}\right]\right]\right].$$

Differentiating $\mathcal{L}_3$ with respect to n_{t+1}, we get

$$\mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} = \mathbb{E}_t \Lambda_{t,t+1}(1-\sigma) + \sigma \mathbb{E}_t \Lambda_{t,t+2} \Omega_{t+2} \Gamma_{t+1}, \tag{1.82}$$

which implies $\Omega_{t+1} = \Delta_{t+1}$ by comparing (1.81) and (1.82).

To get the closed form of the value function V_t, we substitute (1.82) into (1.79)

$$\begin{aligned} V_t(n_t) &= \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} R_{t+1} n_t + \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} \left(R_{k,t+1} - R_{t+1}\right) \phi_t n_t \ldots \\ &+ \mathbb{E}_t \Lambda_{t,t+1} \Omega_{t+1} x_t \left(R_{t+1} - R_{e,t+1}\right) \phi_t n_t. \end{aligned}$$

It is the same as the solution (1.73) derived by the previous method.

1.5.4 Robustness Testing

The result of this paper is robust to the choice of parameters in the real sector and of the size of the financial shock (the standard deviation from 0.5% to 2%). We show that it is also robust to the choice of parameters in the banking sector at reasonable intervals. To be specific, the results hold when κ ranges from 5 to 20, ε from -4 to 0 and θ from 0.15 to 0.35. Details are shown in Table 1.3.

1.5.5 The Risky Steady State

The idea of the risky steady state is from the literature Campbell (1994), Lettau (2003), and Coeurdacier et al. (2011), and de Groot (2013) formally defines the risky steady state. Generally, a general equilibrium model can be described as n non-linear equations with n variables ($\mathbf{x}_t$),

$$\mathbb{E}_t[\mathbf{g}(\mathbf{x}_{t+1})] = \mathbb{E}_t\left[\mathbf{g}(\mathbf{y}_{t+1}^{+}, \mathbf{y}_t, \mathbf{y}_{t-1}^{-})\right] = \mathbf{0}.$$

Table 1.3: Robustness Testing

	Baseline	GKQ	New Policy	Baseline	GKQ	New Policy
Parameter (θ) (0.15 - 0.35)	0.15	0.15	0.15	0.35	0.35	0.35
Capital (K)	216.3214	223.6795	226.3843	201.3784	209.4274	211.7824
Outside Equity Ratio (x)	0.1115	0.1722	0.1761	0.101	0.1572	0.1472
Leverage Ratio (ϕ)	7.8755	7.9848	8.1703	5.9531	6.1835	6.3668
Consumption Equivalent (%)	0	1.15	2.6105	0	0.4891	1.0719
Parameter (ε) (-4 - 0)	-4	-4	-4	0	0	0
Capital (K)	218.6573	226.337	228.0944	204.403	212.3429	214.3057
Outside Equity Ratio (x)	0.2406	0.2874	0.2767	0.0504	0.1124	0.1112
Leverage Ratio (ϕ)	8.3633	8.5311	8.7651	6.2545	6.4687	6.6232
Consumption Equivalent (%)	0	0.6413	1.065	0	0.5183	1.0858
Parameter (κ) (5 - 20)	5	5	5	20	20	20
Capital (K)	211.1076	218.5153	227.3022	205.7405	213.8385	215.9743
Outside Equity Ratio(x)	0.2747	0.3474	0.2826	0.08697	0.1432	0.1481
Leverage Ratio (ϕ)	7.1876	7.3605	8.6845	6.4188	6.6521	6.8011
Consumption Equivalent (%)	0	0.6714	1.0938	0	0.5204	1.4901
Parameter (Ψ) (0.5%- 2%)	0.5	0.5	0.5	2	2	2
Capital (K)	209.419	214.5677	216.6659	201.2497	215.0543	217.5416
Outside Equity Ratio (x)	0.0975	0.1499	0.1496	0.1163	0.1731	0.1586
Leverage Ratio (ϕ)	6.7684	7.0112	7.2102	6.0604	6.2331	6.3487
Consumption Equivalent (%)	0	0.1941	0.7666	0	1.7914	3.5159

where $\mathbf{y}_{t+1}^{+}$ is a vector of n_1 forward-looking variables, $\mathbf{y}_t$ is a vector of n_2 static variables, $\mathbf{y}_{t-1}^{-}$ is a vector of n_3 predetermined variables ($n = n_1 + n_2 + n_3$), and $\mathbf{g}$ is a vector of n non-linear functions. For example, if there is one forward-looking variable y_{t+1}^{+} in the ith equation, we do the second-order approximation around its conditional expectation. So the second-order Taylor expansion of the ith equation ($i = 1, \ldots, n$) is

$$\Phi_i(y_{t+1}^{+}, \mathbf{y}_t, \mathbf{y}_{t-1}^{-}) = \mathbf{g}_i(\mathbb{E}_t y_{t+1}^{+}, \bar{\mathbf{y}}, \bar{\mathbf{y}}^{-}) + \bigtriangledown \mathbf{g}_i \mathbb{E}_t(\mathbf{x}_{t+1} - \bar{\mathbf{x}}) + \frac{1}{2!}\mathbf{g}_i'' \mathbb{E}_t(y_{t+1}^{+} - \mathbb{E}_t y_{t+1}^{+})^2 + o(\cdot) \simeq 0, \tag{1.83}$$

where the variables with 'bar' are those in the steady state, $\bigtriangledown \mathbf{g}_i(\cdot)$ is the gradient, and $\mathbf{g}_i''(\cdot)$ is the second derivative at the point $\mathbb{E}_t y_{t+1}^{+}$. We can see that $\mathbb{E}_t(y_{t+1}^{+} - \mathbb{E}_t y_{t+1}^{+})^2$ is the conditional variance of y_{t+1}^{+} and according to the risky steady state, it is not zero. In other words, the term $\mathbb{E}_t(y_{t+1}^{+} - \mathbb{E}_t y_{t+1}^{+})^2$ is the key difference between the risky steady state and the deterministic steady state.

Following Coeurdacier et al. (2011) and GKQ, we use the iteration method to solve the risky steady state, and it can be summarized as follows: (i) we solve the deterministic steady state of the model, and evaluate the second moments of the forward-looking variables in a linearized system around the deterministic steady state; (ii) use the derived second moments to update the steady state values in the system (1.83) with second order terms; (iii) derive the linearized system around the updated steady state and compute the second moments of this system, substitute the second moments into (1.83) and solve for $\bar{\mathbf{x}}$ to update the steady state. We keep iterating and updating the steady state values and the second moments until they satisfy the steady state system simultaneously.

Besides, de Groot (2013) combines Schmitt-Grohe and Uribe's (2004) simple way to compute the second-order approximate system around the deterministic steady state. In de Groot (2013), the second-order approximation system can be used to compute the risky steady state with lower computational costs.

1.5.6 Welfare and Consumption Equivalent

Welfare $\mathcal{W}_t$ in the model is the steady state of the representative household's lifetime utility

$$\mathcal{W}_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} U\left(C_\tau, C_{\tau-1}, L_\tau\right). \tag{1.84}$$

We take the second-order approximation of the utility function around the risky steady state, so the volatilities of consumption and labor are included. If we write (1.84) in a recursive form,

$$\mathcal{W}_t = U\left(C_t, C_{t-1}, L_t\right) + \beta \mathbb{E}_t \mathcal{W}_{t+1}.$$

In the risky steady state, we have

$$\mathcal{W} = \frac{U(C,C,L) + \beta \mathcal{M}(C,L)}{1-\beta},$$

where $\mathcal{M}(C,L)$ is the second moments of C and L.

We use consumption equivalent to compare the welfare between different policy rules. Denote Γ as the percentage increase of consumption needed for the baseline model to reach the same level of welfare under macroprudential policies. Denote $\mathcal{W}^{GKQ}$

as the welfare under the GKQ rule and Γ^{GKQ} as the consumption equivalent, and we have

$$\mathcal{W}^{GKQ} = \frac{U\left((1+\Gamma^{GKQ})C^B, (1+\Gamma^{GKQ})C^B, L^B\right) + \beta\mathcal{M}(C^B, L^B)}{1-\beta}, \tag{1.85}$$

where C^B and L^B are the risky steady state of consumption and labor in the baseline model.

1.5.7 Impulse Responses under the Constant Subsidy Rule

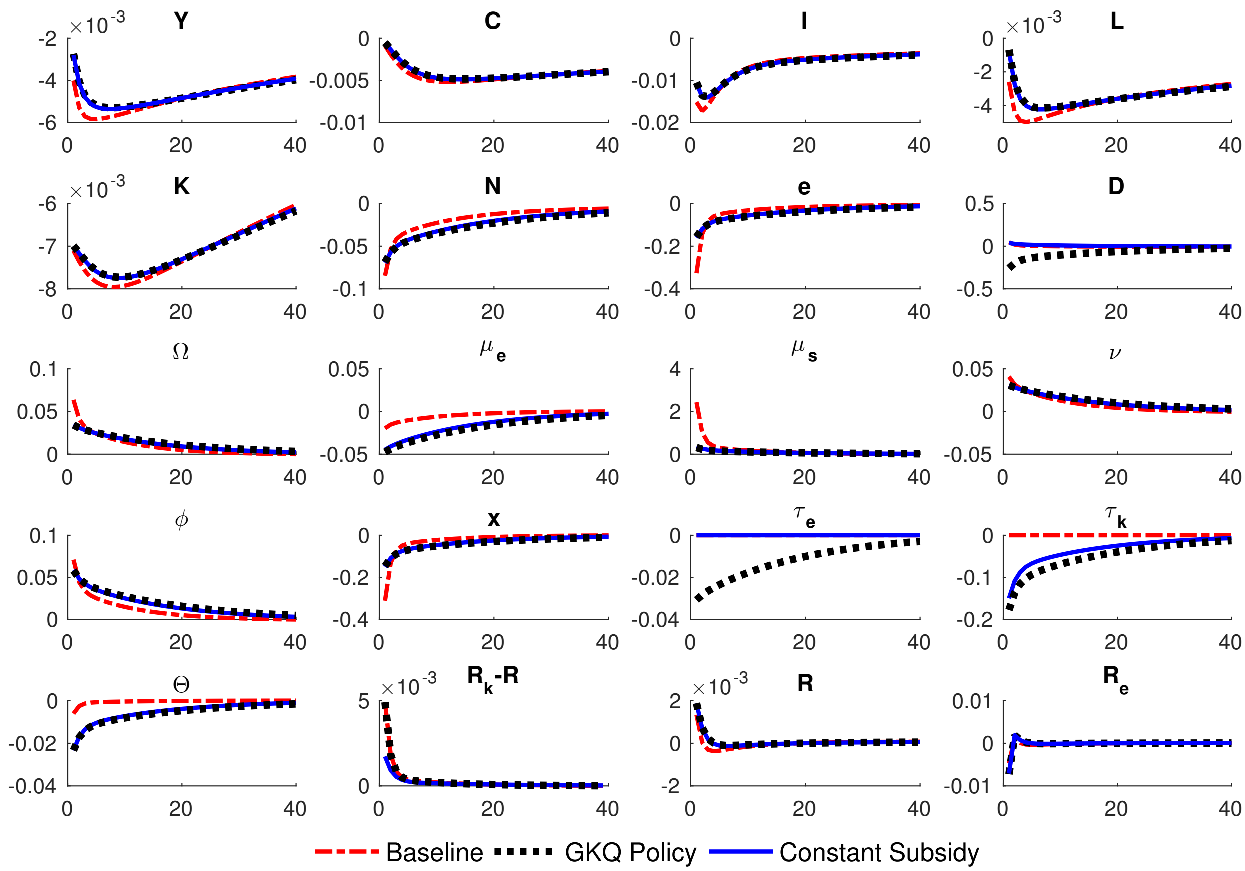


Figure 1.4: Baseline Model, GKQ Policy and Constant Subsidy Policy

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Chapter 2

Stationarity of Econometric Learning with Bounded Memory and a Predicted State Variable

Abstract

In this paper, we consider a model where producers set their prices based on their prediction of the aggregated price level and an exogenous variable, which can be a demand or a cost-push shock. To form their expectations, they use OLS-type econometric learning with bounded memory. We show that the aggregated price follows the random coefficient autoregressive process and we prove that this process is covariance stationary.

Keywords: Econometric Learning, Bounded Memory, Random Coefficient Autoregressive Process, Stationarity

2.1 Introduction

Econometric Learning was designed to model the forecast of the future economic variables in forward looking models. In contrast to the Rational Expectations Theory, which imposes a very strong assumption that the agents know the structure of the model, Econometric Learning only assumes that agents behave as professional econometricians. They collect the available data and use OLS regression to produce the forecast. As more data becomes available, this econometric forecast often converges to the rational expectations equilibria (Sargent, 1993). Although econometric learning relaxes many assumptions of the rational expectations mechanism, we think that one of them could still be too strong. In particular, it assumes that agents have access to the entire history of the variables, and they use all of them to form the forecast. Not only does that assumption require infinite memory, it also neglects the cost of data collection and processing.

Several papers facilitate the assumption of infinite memory and consider the case when the memory is bounded (for a survey, see Chevillon and Mavroeidis, 2014). However, the majority of the results are proven for non-stochastic models (Evans and Honkapohja, 2000). The only exception known to us is Honkapohja and Mitra (2003) who investigate learning with bounded memory in a stochastic environment. However, they consider a very special case of learning the intercept parameter, and their model does not account for the possibility of using some exogenous independent variables when the expectation is formed.

This paper picks up the research from Honkapohja and Mitra (2003) and explores the dynamic properties of econometric learning with bounded memory in a stochastic environment. We expand that paper by adding a stochastic exogenous variable which can be used for econometric forecasts.

The introduction of stochastic independent variable makes the mathematical framework more complex as compared to Honkapohja and Mitra (2003) where the model evolves according to **a** simple autoregressive process (AR). In this paper**,** the model is more complex since the transition matrix has random coefficients (the random coefficient autoregressive model, $RCAR$, as in Nicholls and Quinn, 1982). It is also more complex than Conlisk (1974), since our transition matrices are autocorrelated. Nevertheless, we proved the stationarity of the model. In addition, we formulate a sufficient condition for stationarity which can be more generally applied in the RCAR literature.

This paper is structured as follows. In Section 2.2 we present the model and introduce OLS-type learning with finite memory. In Section 2.3 we prove that the $RCAR$ process of price movement is covariance-stationary. Section 2.4 concludes the paper.

2.2 The model

We consider a model where producer j sets the current price $p_t(j)$ depending on the expected aggregated level of price p_t^e and the exogenous but not completely observable state variable $\widetilde{w}_t$:

$$p_t(j) = \alpha_1 + \beta_1 p_t^e + \delta \widetilde{w}_t \tag{2.1}$$

where α_1, δ are known constant parameters and $\widetilde{w}_t$ is the estimated value of the exogenous cost push shock which can negatively affect the profit. The cost push shock w_t is not observed in period t; however, every producer has access to the historical data of its past realisation of $\{w_s\}$.

This model is very similar to the cobweb model as presented in Kaldor (1934), Ezekiel (1938) and more recently in Evans and Honkapohja (2003). It is known to be stable when $|\beta_1| < 1$. We will restrict our analysis to this particular case. In equilibrium, each producer sets the same price, that is $p_t = p_t(j)$.

2.2.1 OLS Learning

As w_t is the only state variable, the producer expects the aggregated price to depend on the variable

$$p_t = \alpha_2 + \beta_2 w_t, \tag{2.2}$$

where α_2 and β_2 are unknown parameters with producer estimates based on available historical data $\{p_s, w_s\}$. The price expectation is then

$$p_t^e = \hat{\alpha}_{2,t-1} + \hat{\beta}_{2,t-1}\widetilde{w}_t \tag{2.3}$$

where $\hat{\alpha}_{2,t}$ and $\hat{\beta}_{2,t}$ are estimated coefficients and $\widetilde{w}_t$ is a proxy for w_t. The classical OLS-type learning model assumes that agents forecast future prices by running the OLS regression using equation (2.2) and that at time t, the available information set consists of the entire history of prices and the exogenous state variable $\{p_s, w_s\}_{s=0}^{t-1}$. Coefficients $\hat{\alpha}_{2,t}$ and $\hat{\beta}_{2,t}$ are OLS estimators on the information set $\{p_s, w_s\}_{s=0}^{t}$.

2.2.2 Learning with Bounded Memory

Learning with bounded memory in our paper simply means that the agent is only using a limited number of observations T to form expectations.[1] The forecast will be made using the same OLS algorithm as in the classical case (2.3); however, we assume that only a finite set of historical data, $\{p_s, w_s\}_{s=t-T}^{t-1}$, is used to estimate the coefficients. Consequently, the estimators $\hat{\alpha}_{2,t}$ and $\hat{\beta}_{2,t}$ are defined as follows:

$$\begin{aligned}
\hat{\beta}_{2,t-1} &= \frac{\sum_{i=1}^{T}\left[(w_{t-i} - \bar{w}_{t-1})(p_{t-i} - \bar{p}_{t-1})\right]}{\sum_{i=1}^{T}\left[(w_{t-i} - \bar{w}_{t-1})^2\right]}, && (2.4)\\
\hat{\alpha}_{2,t-1} &= \bar{p}_{t-1} - \hat{\beta}_{2,t-1}\bar{w}_{t-1}, && (2.5)\\
\bar{w}_{t-1} &= \frac{1}{T}\sum_{i=1}^{T} w_{t-i}, && (2.6)\\
\bar{p}_{t-1} &= \frac{1}{T}\sum_{i=1}^{T} p_{t-i}. && (2.7)
\end{aligned}$$

Finally, as the agents cannot observe the realization of w_t at the time when they set their prices, the forecast $\widetilde{w}_t$ is used. The forecast is based on available historical data $\{w_s\}_{s=t-T}^{t-1}$, and consists of the weighted sum as in Mitra and Honkapohja (2003).

[1]This is similar to Honkapohja and Mitra (2003) **where** a simplified version of the model without state variable is considered.

Formally, $\widetilde{w}_t$ can be written as

$$\widetilde{w}_t = \sum_{i=1}^{t-1} \gamma_{i,t} w_{t-i}, \tag{2.8}$$

where $\gamma_{i,t}$ is the expected probability that $w_t = w_{t-i}$ and therefore,

$$\sum_{i=1}^{t-1} \gamma_{i,t} = 1. \tag{2.9}$$

Our set up covers an extensive range of models. For example, if w_t follows a Markov process with high persistency, the best prediction for w_t is w_{t-1}. In this case, $\gamma_{1t} = 1$, and $\gamma_{it} = 0$ for $i > 1$. In particular, for $T = 2$, $\gamma_1 = 1$, $\gamma_2 = 0$, the price p_t follows a simple autoregressive process with $p_t^e = p_{t-1}$. If w_t is $i.i.d.$ distributed, the best proxy for w_t might be $\bar{w}_{t-1}$. In this case, $\gamma_{i,t} = \frac{1}{T}$, and the price p_t follows the $AR(T)$ process with $p_t^e = \bar{p}_{t-1}$. Our model will also work if $\gamma_{i,t}$ corresponds to precautionary predictors with larger weights attached to the worse realisations as in the Robust Control or The Ambiguity Aversion theories.

The complete model consists of (2.1), (2.3), (2.8), (2.4), (2.5), (2.6) and (2.7). Our aim is to show that p_t is stationary for all $T > 1$.

First, we show that the aggregated price p_t follows a Random Coefficient Autoregressive ($RCAR$) process.

Proposition 2 *The actual price follows an autoregressive process of order T with random coefficients as in (2.10)*

$$p_t = \alpha_1 + \beta_1 \left(\sum_{i=1}^{T} Z_{i,t} p_{t-i} \right) + \delta \widetilde{w}_t \tag{2.10}$$

where

$$Z_{i,t} = \frac{1}{T} + \frac{(w_{t-i} - \bar{w}_{t-1}) \left(\left(\sum_{i=1}^{T} \gamma_{i,t} \left(w_{t-i} - \bar{w}_{t-1} \right) \right) \right)}{\sum_{i=1}^{T} \left[(w_{t-i} - \bar{w}_{t-1})^2 \right]}. \tag{2.11}$$

2.3 Stationarity of Bounded Memory Learning

Proposition 2 allows us to write our model in the $RCAR$ representation.

$$y_t = \varepsilon_t + M_t y_{t-1}, \tag{2.12}$$

where $M_t = \beta_1 Z_t + S$ and S is **a** lower shift matrix,

$$y_t = \begin{pmatrix} p_t \\ p_{t-1} \\ .... \\ p_{t-T+1} \end{pmatrix}, \; Z_t = \begin{pmatrix} Z_{1,t} & Z_{2,t} & ... & Z_{T,t} \\ 0 & 0 & ... & 0 \\ .... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix} \text{ and } \varepsilon_t = \begin{pmatrix} \alpha_1 + \delta \widetilde{w}_t \\ 0 \\ .... \\ 0 \end{pmatrix}.$$

We begin our investigation of stationarity of the model (2.12) by setting up additional properties of coefficients $Z_{i,t}$.

Lemma 3 *For any realisation of* w_t, *i)* $\sum_{i=1}^{T} Z_{i,t} = 1$ *and ii)* $\sum_{i=1}^{T} Z_{i,t}^2 \leq 1$.

Proof. It is convenient to define $k_{i,t} = \frac{(w_{t-i} - \bar{w}_{t-1})}{\left(\sum_{i=1}^{T}(w_{t-i-1} - \bar{w}_{t-1})^2\right)^{\frac{1}{2}}}$. Then**,** according to (2.11), $Z_{i,t} = \frac{1}{T} + k'_{i,t}\gamma'_t k_t$, where $k_{i,t}$ can be any number with the following restrictions

$$\sum_{i=1}^{T} k_{i,t} = 0, \tag{2.13}$$

$$\sum_{i=1}^{T} k_{i,t}^2 = 1. \tag{2.14}$$

Now we can compute:

$$\sum_{i=1}^{T} Z_{i,t}^2 = \sum_{i=1}^{T} \left(\frac{1}{T} + k_{i,t}\gamma'_t k_t \right)^2 = \frac{1}{T} + \left(\gamma'_t k_t\right)^2. \tag{2.15}$$

Maximisation of (2.15) subject to constraints (2.13) and (2.14) implies $\left(\gamma' k_t\right)^2 = \sum_{i=1}^{T} \gamma_{i,t}^2 - \frac{1}{T}\left(\sum_{i=1}^{T} \gamma_{i,t}\right)^2$ at the maximum. Evaluating (2.15) entails $\sum_{i=1}^{T} Z_{i,t}^2 = \frac{1}{T} + \left(\gamma' k_t\right)^2 \leq \frac{1}{T} + \left(1 - \frac{1}{T}\right) = 1.$ ■

Lemma 3 also implies that $|Z_{i,t}| < 1$. For further discussion, it is convenient to define a random matrix $G_{t,n} = \prod\limits_{k=1,n} (M_{t-k})$. To show that it is finite, we will first establish the boundaries for every element of such a matrix.

Proposition 4 *Consider matrix M_{t-k} such that $M_{t-k} = \beta_1 Z_{t-k} + S$, where S is a lower shift matrix, $|\beta_1| < 1$ and element $z_{i,j}$ of matrix Z_{t-k} satisfies*

$$\begin{aligned} |z_{1,j}| &\leq 1, \\ z_{i,j} &= 0, \quad if \quad i > 1. \end{aligned}$$

Then, for any memory length T and $\widetilde{\beta} \in (\beta_1, 1)$, there exists a finite boundary c_T such that for any n, every element of the product of n matrixes, $G_{t,n}$, is bounded in absolute value by $c_T \widetilde{\beta}^n$ and therefore

$$|G_{t,n}| < c_T \widetilde{\beta}^n J,$$

where J is a $T \times T$ matrix of ones.

Proof. See Appendix 2.5.1. ■

Having established these results, we could investigate the stationarity of y_t by proving the existence of the unconditional expectations $E[y_t]$ and $E[y_t y_t']$.

Proposition 5 *Process (2.12) is covariance stationary if there exist unconditional expectations of $E[|\varepsilon_t|]$ and $E[|\varepsilon_t \varepsilon_t'|]$.*

Proof. To prove stationarity, we will iterate the backward expression (2.12):

$$y_t = \varepsilon_t + M_t y_{t-1} = \varepsilon_t + M_t \varepsilon_{t-1} + M_t M_{t-1} y_{t-1} = \sum_{k=0}^{\infty} G_{t,k} \varepsilon_{t-k}. \tag{2.16}$$

First, we will prove that the expectation of y_t is finite by applying proposition 4:

$$E[y_t] < E[|y_t|] < E\left[\sum_{k=0}^{\infty} |G^{t,k}| |\varepsilon_{t-k}|\right] < J c_T E\left[|\varepsilon_t| \sum_{k=0}^{\infty} \widetilde{\beta}^n\right] = \frac{c_T}{1-\widetilde{\beta}} J\left[E|\varepsilon_t|\right].$$

Thus, we have proved that $E[|y_t|]$ is finite if $E[|\varepsilon_t|]$ exists. To complete the proof, we need to show that $E[y_t y_t']$ is also finite:

$$y_t y_t' = \varepsilon_t \varepsilon_t' + M_t y_{t-1} \varepsilon_t' + \varepsilon_t y_{t-1}' M_t' + M_t y_{t-1} y_{t-1}' M_t'. \tag{2.17}$$

We iterate it backwards to obtain:

$$y_t y_t' = \sum_{k=0}^{\infty} G_{t,k} \left[\varepsilon_{t-k} \varepsilon_{t-k}' + M_{t-k} y_{t-k-1} \varepsilon_{t-k}' + \varepsilon_{t-k} y_{t-k-1}' M_{t-k}' \right] G_{t,k}.$$

Finally, we will show that the expectations of the absolute value of the product are bounded[2]:

$$\begin{aligned} E\left[|y_t y_t'|\right] &= E \sum_{k=0}^{\infty} \left[|G_{t,k}| \left[|\varepsilon_{t-k} \varepsilon_{t-k}'| + |M_{t-k}||y_{t-k-1}||\varepsilon_{t-k}'| + |\varepsilon_{t-k}||y_{t-k-1}'||M_{t-k}'| \right] |G_{t,k}| \right] \\ &< c_T^2 J \left(E\left[|\varepsilon_t \varepsilon_t'|\right] + J E\left[|y_t|\right] E\left[|\varepsilon_t'|\right] + E\left[|\varepsilon_t|\right] E\left[|y_t'|\right] J \right) J \sum_{k=0}^{\infty} \widetilde{\beta}^{2k} \\ &= c_T^2 J \left(E\left[|\varepsilon_t \varepsilon_t'|\right] + J E\left[|y_t|\right] E\left[|\varepsilon_t'|\right] + E\left[|\varepsilon_t|\right] E\left[|y_t'|\right] J \right) J \frac{1}{1 - \widetilde{\beta}^2}. \end{aligned}$$

■

Another interesting implication of Proposition 4 is that the spectral radius of M_t is smaller than one.

Lemma 6 *For any realization of the stochastic matrix M_t, its eigenvalues are less than one in absolute value.*

Proof. Consider $G_n = (M_t)^n$. Applying proposition 4 we can claim that $|G_n| < c_T \widetilde{\beta}^n J$:

$$\lim_{n \to \infty} (M_t)^n = 0$$

which is necessary and sufficient for eigenvalues to be less than one in absolute value.

■

[2]We use that $|M_t| < J$.

2.4 Conclusion

In this paper, we have investigated properties of econometric (OLS-type) learning with a bounded memory. We have shown that the eigenvalues of the transition matrix lie in the unit circle for any length of memory T. Furthermore, we have found that the price p_t follows a covariance stationary process. Our results could be tested in a DSGE framework, similarly to Berardi and Galimberti (2014).

2.5 Technical Appendix

2.5.1 Proof of Proposition 4

For any memory length T, and constant $\widetilde{\beta} > \beta$, there exists a boundary c_T such that every element of the product of n matrices M_t is bounded in absolute value by $c_T\widetilde{\beta}^n$:

$$|G_{t,n}|_{ij} = \left|\prod_{i=1,n}(M_{t-i})\right|_{ij} < c_T\widetilde{\beta}^n, \tag{2.18}$$

where the matrix M_t can be represented as follows

$$M_t = \beta Z_t + S, \tag{2.19}$$

where Z_t has the form of

$$Z_t = \begin{pmatrix} Z_{1,t} & Z_{2,t} & ... & Z_{T-1,t} \\ 0 & 0 & ... & 0 \\ .... & ... & ... & ... \\ 0 & 0 & ... & 0 \end{pmatrix}, \tag{2.20}$$

where each element $Z_{i,t}$ is smaller than 1 in absolute value, $|Z_{i,t}| < 1$; and S is the lower shift matrix

$$S = \begin{pmatrix} 0 & 0 & ... & 0 & 0 \\ 1 & 0 & ... & 0 & 0 \\ 0 & 1 & ... & 0 & 0 \\ .... & ... & ... & ... & ... \\ 0 & 0 & ... & 1 & 0 \end{pmatrix}. \tag{2.21}$$

Proof. First we will compute the product

$$G = \prod_{i=1,n}(M_{t-i}) = (\beta_1 Z_{t-1} + S)(\beta_1 Z_{t-2} + S)....(\beta_1 Z_{t-n} + S) \tag{2.22}$$

using the property of matrix S. For any matrix A, the first row of SA is zero. Moreover, if the first k rows of A are zeros, then the first of $k+1$ rows of SA are also zeros.

To compute the product (2.22) we need to sum up the products of n matrixes, each of them is either Z or S. However, if S appears more than $T-1$ times, the product is zero. Therefore, we can restrict our attention to only those cases when S appears less than T times.

The number of products with S being exactly on k places is $n!/k!/(n-k)!$ and therefore, the total number of non-zero products is less than $n!/((T-1)!(n-T+1)!) \times (T-1)$. Moreover, we can claim that every component is a matrix with elements less than $(\beta z)^{n-T}$, where $z = \max_i |Z_{i,t}| \leq 1$. Therefore, every element of

$$[(\beta_1 Z_{t-1} + S)(\beta_1 Z_{t-2} + S) \dots (\beta_1 Z_{t-n} + S)]_{ij} < \frac{n!}{(T-2)!(n-T+1)!} \beta^{n-T} < n^T \beta^{n-T}.$$

Consider the sequence $\{a_n\}$, defined as $a_n = n^T \beta^{n-T}$,

$$\frac{a_{n+1}}{a_n} = \left(\frac{n+1}{n}\right)^T \beta.$$

Let $\widetilde{\beta} \in (\beta, 1)$, then we can find $n^*(\widetilde{\beta}, T, \beta)$, such that for any $n > n^*$,

$$\frac{a_{n+1}}{a_n} = \left(\frac{n+1}{n}\right)^T \beta < \widetilde{\beta}, \tag{2.23}$$

in particular

$$n^* = ceil\left(\left[\left(\frac{\widetilde{\beta}}{\beta}\right)^{1/T} - 1\right]^{-1}\right).$$

It follows from (2.23) that for any positive k

$$a_{n^*+k} < \widetilde{\beta}^k a_{n^*} = \widetilde{\beta}^k n^{*T} \beta^{n^*-T}. \tag{2.24}$$

To complete the proof we define c_T

$$c_T = \max_{n \le n^*} \left(a_n \widetilde{\beta}^{-n} \right) .$$

■

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Chapter 3

A comment on: "Capital regulation and monetary policy with fragile banks"

Abstract

This paper comments on Angeloni and Faia (2013, Journal of Monetary Economics), a dynamic stochastic general equilibrium model with a risky banking sector. We identify the sources of inefficiency in the model and disentangle the channels through which banks choose a high level of leverage. We explain that their assumptions that generate banks over-borrowing feature lead to the return on assets and the bankruptcy probability that are unrealistically high. Next, we modify the model by incorporating the banking sector of Gertler and Karadi (2011) into the AF model and show that the calibration result improves.

3.1 Introduction

Angeloni and Faia (2013), AF henceforth, incorporate a potential bank failure into a dynamic stochastic general equilibrium (DSGE) model, proposing a framework in which the interaction between monetary policy and bank risk can be analyzed. In short, the following is what their model can generate. When the economy is hit by positive productivity shock, output goes up, inflation goes down, and a Taylor-rule type policy lowers the nominal interest rate, as can happen in a standard DSGE model. The lower nominal interest rate then encourages banks to borrow more, raising bank leverage, aggregate investment, and output. This is the amplification of a standard DSGE mechanism. Similarly, when the economy is hit by an expansionary monetary policy shock, the lowered nominal interest rate encourages banks to borrow and invest more, again leading to amplification. In the meantime, the higher leverage by banks leads to higher chance of bank failure and potential welfare loss by default cost.

Novel as it is, AF do not clearly tell how excessive bank leverage can arise in their model, nor whether the equilibrium bank leverage in their model is suboptimal. In this article, we demonstrate that their equilibrium bank leverage is indeed suboptimal. We identify the source of inefficiency in their model and disentangle the mechanism through which banks' excessive borrowing arises. We find that this very mechanism makes it difficult to obtain desirable calibration outcome. Next, we modify the AF model by taking Gertler and Karadi's (2011) way of modeling the banking sector. We explain the source of inefficiency in this modification and study the calibration result in comparison with the original AF model.

There are a few minimum required features for a desirable model. First, banks borrowing to some extent should benefit the society but too much borrowing should harm the society so that the socially optimal bank leverage is "interior" (that is, neither zero nor infinity). For this, AF assume that physical capital can be funded only by banks and that banks cannot issue stocks: that is, investment must be funded by internal equity and deposits only. Also, if the bank cannot fully repay depositors, there is default cost. These two combined give a positive role to a small, but not too high, bank leverage.

Second, the equilibrium level of leverage that is chosen by individual banks should be different from what the social planner would choose subject to the same constraints. The choice of the bank's objective turns out to be a delicate issue. There must be a source of constrained inefficiency, or in other words, the bank must fail to internalize something when it selects its leverage. One possible story is that the bank maximizes the stockholder's payoff, and in the presence of deposit insurance or implicit government guarantee, the degree of leverage chosen by the bank does not have a direct effect on the bank's borrowing cost, allowing it to take an excessive risk. In such a story, what the bank fails to internalize is the potential cost of bank failure borne by tax-payers. AF take a somewhat different approach without explicitly stating the source of inefficiency. They assume that when the bank chooses the leverage, the objective is the rate of return to bank asset minus a state-dependent management fee. Once the model induces banks to choose too high leverage in equilibrium, it follows that there is a role of capital requirement in the model, allowing for a normative analysis for the optimal capital regulation.

Once the channels through which the bank takes excessive leverage relative to the socially optimal level are made clearer, we have a better interpretation of calibration results. We claim that the state-dependent management fee plays an important role in the AF model and this fee must be sufficiently high for the model to work. When the model is calibrated, this leads to a low return to bank capitalists and hence a very low level of investment and a very high level of return on investment.

Next, we consider a modification by modeling the banking sector in the way of Gertler and Karadi (2011) (we call it 'GK' hereafter). The source of inefficiency now comes from the banks maximizing the shareholder's payoff. When banks choose their leverage ratio, they fail to internalize the potential default cost on lenders, resulting in an excessively high level of bank leverage. At the same time, the model could achieve a neither zero nor infinite level of banks borrowing because of the financial constraint in the GK model: the bank's ability to absorb funds is limited by the internal bank equity within the bank. Consequently, when the model is calibrated, the return to bank capitalists is not as low as that in AF model and therefore, the return on investment can be calibrated to fit the empirical data. Meanwhile, the new source of inefficiency retains the over-borrowing feature. The comparison between the AF model and the modified model highlights how different ways of creating inefficiency lead to different features in calibration.

Because we never know the true macroeconomic mechanism, all macroeconomic models are in a sense "geocentric models". Geocentric models are still useful in making possible channels more visible and perhaps also in forecasting phenomena that the model was originally designed to explain. In the field of asset pricing, for example, because the goal is mainly about forecasting, the model can be evaluated based on how many moments and correlations can be matched to those of data. Geocentric models can be harmful, however, when they are used to get normative implications such as desirable policies. That is why we should at least be clear about the source of inefficiency, and about how a policy counteracts it. Comparing different models that drive similar quantitative results is, therefore, a meaningful line of research in quantitative macroeconomics. This paper is one such attempt.

The rest of the paper is as follows: Section 2 describes the AF model. Section 3 discusses the source of inefficiency and shows calibration results in the AF model. Section 4 shows the modification of the model and Section 5 shows its simulation results. Section 6 concludes.

3.2 The Model

This section briefly describes the original model from Angeloni and Faia (2013). There are six sectors in the model: households, goods producers, capital producers, banks, the central bank, and the government. The banking sector is the novel part in AF, and the other sectors are standard in dynamic stochastic general equilibrium (DSGE) frameworks.

3.2.1 Households

A representative household has a continuum of members with mass 1, where a fraction f of them are workers and the remaining $1-f$ are bank capitalists[1]. Workers supply labor to firms to earn wages, and bank capitalists work in a financial intermediary and get a lump-sum payment of bank dividends when they exit the banking sector. Household maximizes the following discounted sum of utilities:

$$\mathbb{E}_t \sum_{\tau=t}^{+\infty} \beta^{\tau-t} U(C_\tau, N_\tau) = \sum_{\tau=t}^{+\infty} \beta^{\tau-t} \left[\frac{C_\tau^{1-\sigma}-1}{1-\sigma} + \eta \ln(1-N_\tau) \right], \tag{3.1}$$

where $\mathbb{E}_t(\cdot)$ denotes the conditional expectation given the information at time t, C_t is consumption, N_t is labour hours, β is the time discount factor, σ is risk aversion and η is the weight of labor disutility to the utility of consumption. The budget constraint is given by

$$P_t(C_t+T_t) + D_t = W_t N_t + R_{t-1}(1-\zeta_t)D_{t-1} + \Pi_t + \Theta_t, \tag{3.2}$$

where P_t is the price of aggregate consumption goods, T_t is the tax that households pay to the government, D_t is deposits, W_t is nominal wage, Θ_t is the net income transferred from goods producers, Π_t is the payment received from the banking sector, R_t is the contractual return of deposits from time t to $t+1$, and ζ_t is the expected loss incurred by one unit of deposits due to possible bank runs. In every period, household chooses consumption, labor supply and deposits (C_t, L_t, D_t) to maximize (1) given (P_t, W_t, R_t). The optimality condition gives the Euler equation and labor supply function:

$$\mathbb{E}_t \left[\beta(1-\zeta_{t+1}) \frac{R_t}{\pi_{t+1}} \frac{C_t^\sigma}{C_{t+1}^\sigma} \right] = 1, \tag{3.3}$$

[1] In every period, some workers become bank capitalists, and some bank capitalists exit the banking sector to become workers. The fractions of workers and bank capitalists are constant over time.

$$w_t \equiv \frac{W_t}{P_t} = \eta \frac{C_t^{\sigma}}{1 - N_t}, \tag{3.4}$$

where w_t is the real wage rate, and $\pi_{t+1} = \frac{P_{t+1}}{P_t}$ is the inflation rate.

3.2.2 Goods Producers

The final good is produced from a variety of differentiated intermediate goods $Y_t(i)$, $i \in [0, 1]$ according to a constant-return-to-scale technology. So the aggregate final good is

$$Y_t = \left(\int_0^1 Y(i)_t^{\frac{\epsilon-1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{3.5}$$

where $\epsilon > 1$ is the elasticity of substitution between various goods. Intermediate good producer i uses labour and capital input to produce differentiated intermediate goods $Y_t(i)$ through a Cobb-Douglas production function:

$$Y_t(i) = A_t K_t(i)^{\alpha} N_t(i)^{1-\alpha}, \tag{3.6}$$

where A_t is the total factor productivity, and $K_t(i)$ and $N_t(i)$ are the capital and labour inputs. Each good producer has monopolistic power so he can set the price of their own products. According to (3.5), producer i faces a downward sloping demand curve:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} Y_t, \tag{3.7}$$

where $P_t(i)$ is the price of good i, and the aggregate price level P_t is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}}. \tag{3.8}$$

They face a convex cost for adjusting the price of their goods (*a la* Rotemberg (1982)):

$$\frac{\vartheta}{2} \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right)^2 P_t, \tag{3.9}$$

where $\vartheta > 0$ is the degree of price rigidity.[2] If $\vartheta > 0$, this results in nominal rigidity in the economy. In every period, each good producer chooses the capital and labour inputs $(K_t(i), N_t(i))$, and sets the price level $P_t(i)$ to maximize the expected discounted real profit:

$$\mathbb{E}_t \sum_{\tau=t}^{+\infty} \Lambda_{t,\tau} \left[\frac{P_\tau(i)}{P_\tau} Y_\tau(i) - \frac{W_\tau}{P_\tau} N_\tau(i) - \frac{Z_\tau}{P_\tau} K_\tau(i) - \frac{\vartheta}{2} \left(\frac{P_\tau(i)}{P_{\tau-1}(i)} - 1 \right)^2 \right], \tag{3.10}$$

subject to the production constraint $A_\tau K_\tau(i)^\alpha N_\tau(i)^{1-\alpha} = \left(\frac{P_t(i)}{P_t}\right)^{-\epsilon} Y_\tau$ for every period. In (3.10), Z_t is the rental rate of capital, and $\Lambda_{t,\tau}$ is the household's stochastic discount factor which takes the form

$$\Lambda_{t,\tau} = \beta^{\tau-t} \frac{C_t^\sigma}{C_\tau^\sigma}. \tag{3.11}$$

Let m_t denote the Lagrange multiplier of the production constraint at period t, which is interpreted as the cost of making an extra unit of production. In symmetric equilibrium, we have $P_t(i) = P_t$, and the optimality condition is given by

$$\frac{W_t}{P_t} = (1-\alpha) m_t A_t \left(\frac{K_t}{N_t} \right)^\alpha. \tag{3.12}$$

$$\frac{Z_t}{P_t} = \alpha m_t A_t \left(\frac{N_t}{K_t} \right)^{1-\alpha}. \tag{3.13}$$

$$(\pi_t - 1)\pi_t = \frac{\epsilon}{\vartheta} \left(m_t - \frac{\epsilon - 1}{\epsilon} \right) Y_t + \mathbb{E}_t \left[\Lambda_{t,t+1} (\pi_{t+1} - 1) \pi_{t+1} \right]. \tag{3.14}$$

After the log-linearzation of (3.14), we get the New Keynesian Phillips curve.

3.2.3 Capital Producers

Competitive capital producers accumulate physical capital subject to adjustment costs:

$$K_{t+1} = (1-\delta) K_t + \xi \left(\frac{I_t}{K_t} \right) K_t, \tag{3.15}$$

[2] Other DSGE models with Rotemberg type nominal rigidity assume that the price adjustment cost is proportionate to the aggregate output level.

where I_t is investment, and $\xi\left(\frac{I_t}{K_t}\right) = \varrho_1 \frac{\left(\frac{I_t}{K_t}\right)^{1-v}}{1-v} + \varrho_2$.[3] The capital producer chooses investment level I_t to maximize the following expected discounted profit subject to (3.15):

$$\mathbb{E}_t \sum_{t=\tau}^{+\infty} \Lambda_{t,\tau} \left(\frac{Z_\tau}{P_\tau} K_\tau - I_\tau \right), \tag{3.16}$$

Let Q_{t+1} denote the Lagrange multiplier of the constraint (3.15), and it is the real price of capital K_{t+1}. The optimal solution is given by

$$Q_{t+1} \xi' \left(\frac{I_t}{K_t} \right) = 1. \tag{3.17}$$

$$\mathbb{E}_t \left[\Lambda_{t,t+1} \frac{Z_{t+1}/P_{t+1} + Q_{t+2} \left(1 - \delta + \xi(\frac{I_{t+1}}{K_{t+1}}) - \xi'(\frac{I_{t+1}}{K_{t+1}}) \frac{I_{t+1}}{K_{t+1}}\right)}{Q_{t+1}} \right] = 1 \tag{3.18}$$

The expression $\frac{Z_{t+1}/P_{t+1} + Q_{t+2}\left(1-\delta+\xi(\frac{I_{t+1}}{K_{t+1}}) - \xi'(\frac{I_{t+1}}{K_{t+1}})\frac{I_{t+1}}{K_{t+1}}\right)}{Q_{t+1}}$ in (3.18) is the real return of holding one unit of capital from time $t+1$ to $t+2$. So the real return of capital from t to $t+1$, denoted as $\frac{R_t^A}{\pi_{t+1}}$, is given by:

$$\frac{R_t^A}{\pi_{t+1}} = \frac{Z_t/P_t + Q_{t+1} \left[(1-\delta) - \xi'(\frac{I_t}{K_t}) \frac{I_t}{K_t} + \xi(\frac{I_t}{K_t}) \right]}{Q_t}. \tag{3.19}$$

3.2.4 The Banking Sector

In AF, it is assumed that banks choose a number of assets (L_t) to invest in, and in equilibrium, $L_t = K_t$ holds. These investment projects pay off one period after and require initial funds. The funds are composed of bank capital (B_t) from bank capitalists and deposits (D_t) collected from households. The bank balance sheet is:

$$Q_t L_t = D_t + B_t, \tag{3.20}$$

[3]The adjustment cost is from Jermann and Quadrini (2012) to capture the pro-cyclicality of asset prices.

where L_t is the number of investment projects and Q_t is the price of the project, D_t is deposits, and B_t is bank capital. Bank managers work for banks in exchange for a state-dependent management fee. They make decisions on the bank capital structure (equivalently, the deposits issuance level) and choose investment projects given the deposit rate R_t and the average return on projects R_t^A. However, the actual return on the project is random: it equals to the average return R_t^A plus a random variable x_{t+1} that follows a uniform distribution over $[-h, h]$. The project goes through two stages before it matures. In the first stage, bank managers and depositors know the actual return of the project $R_t^A + x_{t+1}$, and depositors decide whether to withdraw early. If the depositors do so, bank managers have to liquidate the project before maturity to meet depositors' demand, which causes a constant fraction c of loss on the project $(0 < c < 1)$. The second stage is simply the procedure of early liquidation, or the time for maturity if early liquidation does not happen. Depositors choose to withdraw early when the actual return on the project is too low to be fully repaid. If it happens, the bank manager is forced to liquidate the project early, and the bank becomes insolvent. In the AF model, depositors choose to run only when banks have insolvency problem, and all depositors receive equal payoff. This type of bank runs is *a la* Allen and Gale (1998).[4]

Following is the timeline: at the beginning of time t, the bank chooses the level of deposits they issue (D_t) and funds investment projects given the average return of investment projects R_t^A and the deposit rate R_t. At the first stage, the actual return on the project x_{t+1} is realized. When it is too low to repay to the depositors, depositors run on the bank, and the bank manager liquidates the project before maturity. In this case, the bank is insolvent and exits the banking sector after the liquidation procedure. When the return is high enough to repay depositors, depositors do no run on the bank, and bank capitalists retain their profit as bank capital. Another round of new investment projects start.

[4] In AF, there is no sequential services for withdrawal as in Diamond and Dybvig (1983). Thus, multiple equilibria are not considered. That is to say, for such realization of return on the project that both bank-run and no bank-run equilibrium exist, the no bank-run equilibrium is selected.

In AF, one further assumption is made regarding the bank's liquidation before maturity: the bank manager obtains better knowledge about the liquidation the project. Let λ denote the ratio of the liquidation value of other agents to that of the bank manager, so we have $0 < \lambda < 1$.

In every period, the bank manager decides the capital structure of the bank by choosing the level of deposits D_t to maximize the expected return of capitalists and depositors (we also call them 'outsiders' hereafter). Equivalently, given the bank capital (B_t) in the bank, the bank manager makes the decision on the deposit share d_t which is defined as a ratio of deposits to the value of investment projects:

$$d_t = \frac{D_t}{Q_t L_t}. \tag{3.21}$$

There will be three different cases depending on different realizations of the return on asset $R_t^A + x_{t+1}$, bank balance sheet structure d_t, and the deposit rate R_t from time t to $t+1$.

Case A: run for sure: It happens when the realized payoff of projects is too low for the bank to repay the depositors, that is, $(R_t^A + x_{t+1})Q_t L_t < R_t D_t$ (i.e. $x_{t+1} < R_t d_t - R_t^A$). In this case, depositors run on the bank and the bank manager liquidate the project before maturity. As residual claimants, bank capitalists get nothing. Depositors can get $\lambda(1-c)(R_t^A + x_{t+1})Q_t L_t$ if they liquidate the project by themselves. How the bank manager and depositors split the remainder of the liquidated project $(1-\lambda)(1-c)(R_t^A + x_{t+1})Q_t L_t$ depends on their bargaining powers. For simplicity, it is assumed that they split the remainder equally, so the bank manager gets $(1-\lambda)(1-c)(R_t^A + x_{t+1})Q_t L_t/2$, and depositors get $(1+\lambda)(1-c)(R_t^A + x_{t+1})Q_t L_t/2$. The total return on assets for outsiders is $(1+\lambda)(1-c)(R_t^A + x_{t+1})/2$.

Case B: run only without the bank: The realization of return on project is high enough to repay the depositors if the bank manager extracts funds from the project , but not enough when other agents extract the funds, that is, $\lambda(R_t^A + x_{t+1})Q_t L_t < R_t D_t < (R_t^A + x_{t+1})Q_t L_t$. In this case, the bank manager avoid the run, so depositors

Table 3.1: Payoffs to depositors, bank capitalists and bank managers

	Case A	*Case B*	*Case C*
Realizations of x_{t+1}	$[-h, R_t d_t - R_t^A]$	$[R_t d_t - R_t^A, \frac{R_t d_t}{\lambda} - R_t^A]$	$[\frac{R_t d_t}{\lambda} - R_t^A, h]$
Bank condition	Run for sure	Banks avoid the run	No bank run possibility
Depositors (1)	$\frac{(1+\lambda)(1-c)(R_t^A+x_{t+1})}{2} Q_t L_t$	$R_t D_t$	$R_t D_t$
Capitalists (2)	0	$\frac{(R_t^A+x_{t+1})-R_t d_t}{2} Q_t L_t$	$[\frac{(1+\lambda)(R_t^A+x_{t+1})}{2} - R_t d_t] Q_t L_t$
Bank managers	$\frac{(1-\lambda)(1-c)(R_t^A+x_{t+1})}{2} Q_t L_t$	$\frac{(R_t^A+x_{t+1})-R_t d_t}{2} Q_t L_t$	$\frac{(1-\lambda)(R_t^A+x_{t+1})}{2} Q_t L_t$
(1) + (2)	$\frac{(1+\lambda)(1-c)(R_t^A+x_{t+1})}{2} Q_t L_t$	$\frac{(R_t^A+x_{t+1})+R_t d_t}{2} Q_t L_t$	$\frac{(1+\lambda)(R_t^A+x_{t+1})}{2} Q_t L_t$

get paid fully $R_t D_t$. It is assumed that bank capitalists and the bank manager split the proceeds after deposits payment equally, so they get $[(R_t^A + x_{t+1})Q_t L_t - R_t D_t]/2$, and depositors and bank capitalists get $[(R_t^A + x_{t+1}) + R_t d_t]Q_t L_t/2$ in total. The total return of assets for both is $[(R_t^A + x_{t+1}) + R_t d_t]/2$.

Case C: no run for sure: The realization of the return on project is high enough to repay the depositors even if bank capitalists extract the funds, so $\lambda(R_t^A + x_{t+1})Q_t L_t > R_t D_t$. In this case, depositors get $R_t D_t$ for sure. Bank capitalists can get $\lambda(R_t^A + x_{t+1})Q_t L_t - R_t D_t$ if they extract the funds by themselves, and this is the lower bound of their payoff. Because the bank manager can extract $(R_t^A + x_{t+1})Q_t L_t - R_t D_t$, she and the bank capitalist split the extra return equally. So the bank manager obtains $(1-\lambda)(R_t^A+x_{t+1})Q_t L_t/2$, and bank capitalists get $(1+\lambda)(R_t^A+x_{t+1})Q_t L_t/2-R_t D_t$. The total return of assets for outsiders is $(1 + \lambda)(R_t^A + x_{t+1})/2$. We summarize the payoffs to depositors, bank capitalists and bank managers in Table 3.1 when the realization of the return x_{t+1} lies within different intervals.

Above all, the ex-ante return of assets to outsiders, denoted as $R_t^{Outsider}$, is

$$R_t^{Outsider} = \frac{1}{2h}\int_{-h}^{R_t d_t - R_t^A} \frac{(1+\lambda)(1-c)(R_t^A + x_{t+1})}{2} dx_{t+1}$$
$$+\frac{1}{2h}\int_{R_t d_t - R_t^A}^{\frac{R_t d_t}{\lambda} - R_t^A} \frac{R_t^A + x_{t+1} + R_t d_t}{2} dx_{t+1} + \frac{1}{2h}\int_{\frac{R_t d_t}{\lambda} - R_t^A}^{h} \frac{(1+\lambda)(R_t^A + x_{t+1})}{2} dx_{t+1} \quad (3.22)$$

AF assume that the bank manager chooses the capital structure d_t to maximize the expected return of assets in expression (3.22), and they show that the optimal capital structure is

$$d_t = \frac{1}{R_t}\frac{R_t^A + h}{2 - \lambda + c(1+\lambda)}.[5] \tag{3.23}$$

The optimal d_t satisfies the condition $\frac{R_t d_t}{\lambda} - R_t^A > h$ and hence, *case C* disappears. Overall, the expected return of holding one unit of deposit from time t to $t+1$ is

$$R_t \left(\frac{1}{2h} \int_{-h}^{R_t d_t - R_t^A} \frac{(1+\lambda)(1-c)(R_t^A + x_{t+1})}{2R_t d_t} dx_{t+1} + \frac{1}{2h} \int_{R_t d_t - R_t^A}^{h} dx_{t+1} \right), \tag{3.24}$$

and the probability of bank run happening from time t to $t+1$ is

$$\phi_{t+1} = \frac{1}{2h} \int_{-h}^{R_t d_t - R_t^A} dx_{t+1} = \frac{R_t d_t - R_t^A + h}{2h}. \tag{3.25}$$

AF denotes g_{t+1} as the expected loss conditional on bank run, so $\phi_{t+1} g_{t+1}$ is the expected loss when holding one unit of deposit, and it equals ζ_{t+1} in Eq. (3.3):

$$\zeta_{t+1} = \phi_{t+1} g_{t+1}. \tag{3.26}$$

According to the expression (3.24) and (3.25), we can derive the expression of g_{t+1} as

$$g_{t+1} = 1 - \frac{(1+\lambda)(1-c)}{4}(1 + \frac{R_t^A - h}{R_t d_t}). \tag{3.27}$$

Note that AF assume that the leverage does not affect the bank's cost of borrowing, R_t, which resembles the effect of people's (wrong) expectations for government bailout.

At the end of every period, it is assumed that a fraction θ of bank capitalists remain in the banking sector. Bank capital accumulates from the payoff retained by bank capitalists, so aggregate bank capital evolves as

$$B_{t+1} = \frac{\theta}{\pi_{t+1}} \varrho_{t+1}^{BK} Q_t K_t, \tag{3.28}$$

[5] The interior solution exists when the parameter values satisfy two conditions: $\frac{(1-\lambda)^2}{\lambda} - c(\lambda+1) > 0$ and $\lambda < \frac{1}{2-\lambda+c(1+\lambda)} < 1$. Both conditions hold in the computational experiment when $\lambda = 0.45$, $h = 0.35$ and $c = 0.2$. The derivations of the optimization problem is shown in their online Appendix.

where ϱ_{t+1}^{BK} is bank capitalists' average return of assets.[6] Since there are a lot of investment projects, the average return is given by

$$\varrho_{t+1}^{BK} = \frac{1}{2h}\int_{R_t d_t - R_t^A}^{h} \left(\frac{R_t^A + x_t - R_t d_t}{2}\right) dx_{t+1} = \frac{(R_t^A + h - R_t d_t)^2}{8h}. \tag{3.29}$$

3.2.5 Resource Constraint and Monetary Policy

The government has a balanced budget so that the lump-sum tax is equal to the exogenous government purchase: $T_t = G_t$. Since price adjustment cost and the loss caused by bank runs are paid in real units, the economy's aggregate resource constraint is given by:

$$\left[1 - \frac{\vartheta}{2}(\pi_t - 1)^2\right] Y_t - \Delta_t = C_t + I_t + G_t, \tag{3.30}$$

where Δ_t represents the aggregate cost for bank runs:

$$\Delta_t = \frac{1}{2h}\int_{-h}^{R_t d_t - R_t^A} c(R_t^A + x_{t+1}) Q_t K_t dx_{t+1} = \frac{c}{4h}\left[(R_t d_t)^2 - (R_t^A - h)^2\right] Q_t K_t. \tag{3.31}$$

In labour market equilibrium, Labour supply equals to labour demand, The central bank conducts the monetary policy following a Taylor–type rule:

$$\ln\left(\frac{R_t}{\bar{R}}\right) = (1 - b_r)\left[b_\pi \ln\left(\frac{\pi_t}{\bar{\pi}}\right) + b_Y \ln\left(\frac{Y_t}{\bar{Y}}\right)\right] + b_r \ln\left(\frac{R_{t-1}}{\bar{R}}\right) + \ln\left(\frac{M_t}{\bar{M}}\right), \tag{3.32}$$

where b_r, b_π, b_Y are policy parameters, and M_t is the monetary shock in the Taylor rule. The expansionary monetary policy corresponds to a negative shock of M_t. All the variables in the policy rule are log-deviations from the steady state, and the notation with 'bar' is the steady state of the variable.

Following are the sequence of actions during time period t: *(i)* The capital K_t is given at the beginning of time t. *(ii)* Shocks A_t and M_t realize. *(iii)* Households choose (C_t, N_t, D_t) given (w_t, R_t, ζ_{t+1}), final good producers choose $(K_t, N_t, P_t(i))$ given $(\frac{Z_t}{P_t}, w_t, P_t)$, capital producers choose I_t given $(\frac{Z_t}{P_t}, K_t)$, banks choose d_t given (R_t, R_t^A),

[6] AF treat ϱ_{t+1}^{BK} as net return (p. 315, Eq. (8)). This typo is corrected here.

where $(w_t, \frac{Z_t}{P_t}, R_t^A)$ are determined by the markets and R_t is chosen by the central bank.

In summary, the equilibrium of the model is defined as $(C_t, R_t, D_t, N_t, w_t, Y_t, K_t, \frac{Z_t}{P_t}, \pi_t, I_t, Q_t, R_t^A)$ that satisfy Eqs. (3), (4), (6), (12), (13), (14), (16), (18), (20), (24), (31), (33).

3.3 Discussion of AF Model

In the AF model, it is assumed that investment can be funded by bank capital and deposits only. Households cannot lend to capital producers directly, so the funds that are lent to the capital producers must go via banks. In this case, a higher level of bank borrowing implies a higher level of investment and therefore, higher output and consumption. Other things equal, it improves welfare when banks issue more deposits. However, an excessively high level of borrowing can also do harm to the welfare. As the level of borrowing goes beyond a certain point, it is possible that banks cannot fully repay the depositors when the realization of outcome in investment projects is very low. The early liquidation cost causes a loss of resource in the economy and lowers the welfare. Under such circumstances, the higher the borrowing level is, the higher the probability of bank runs and higher liquidation cost borne by households. From the above, the issuance of some but not too much deposits can improve the welfare, and the socially optimal level of bank leverage trades off these two opposite effects of borrowing.

The level of bank borrowing in the AF framework is highly influenced by two assumptions: First, bank managers maximize the combined return of bank capitalists and depositors; Second, bank managers have bargaining power to take some payoff from bank assets. The first assumption ensures that the borrowing level decided by bank managers is finite,[7] and the second guarantees that bank managers choose a level of deposit ratio such that the probability of bank runs is positive. The reasons are as

[7] If bank managers maximize the payoff level instead of the return of payoff, the bank managers' optimal decision is to borrow an infinite amount of deposits. The details of derivation are shown in Appendix 3.6.4.

follows.

When bank managers have advantage in liquidating projects, they either *(i)* take a share of payoff from depositors in bank-run case or *(ii)* receive a share of proceeds from bank capitalists in no-bank-run case. When the deposit ratio is as low as $d_t < \frac{R_t^A - h}{R_t}$, banks can always fully repay the depositors, so *(i)* is zero but *(ii)* is large; When the deposit ratio is high enough so that there is possibility of bank runs (i.e., $d_t > \frac{R_t^A - h}{R_t}$), a higher level of d_t enlarges the region of *Case A* and reduces the region of *Case B*, so the bank manager gets more in *(i)* and less in *(ii)*. However, bank managers could get a larger fraction of bank assets in *(ii)* than in *(i)* because the liquidation advantage which helps the bank to avoid runs in *Case B* allows bank managers to get higher payoff. In this case, a higher level of d_t reduces the payoff of bank managers and raises the return of the outsiders. As a result, when bank manager maximizes the return of outsiders, they choose the deposit ratio that opens the possibility of bank runs.[8]

Now we can identify sources of inefficiency in the AF model. When bank managers set the combined return of both bank capitalists and depositors as objective, they take into account the positive effect of a higher level of borrowing on the welfare, but only to a certain extent as they do not take their own payoff into account. By considering the return of depositors, they include the negative effect of the early liquidation cost on the return of investment projects. This somehow includes the negative effect on the aggregate resource in the economy, but only partially. The total bank assets are lent to firms to rent capital and according to (3.6), the bank assets, which equals to the level of physical capital, contribute a constant share $\alpha < 1$ to the final goods production. From (3.30), however, the liquidation cost is deducted from the final output in the resource constraint. That is to say, when bank runs may occur, the negative effect of the early liquidation cost on the aggregate resource is more severe than that on bank assets. As a result, bank managers do not *fully* internalize its negative effect on the welfare and undervalue the cost of bank runs. Thus, they choose a higher level of deposit level than

[8]If bank managers include themselves in their objective or equivalently if $\lambda = 1$, the possibility of bank run is never optimal. The optimal leverage in that case is given by (3.37) in Section 3.3.1.

the socially optimal level, which leads to a higher probability of bank runs and incurs a higher waste of liquidation cost. This is one source of inefficiency.

Besides, the bank capital structure in AF yields another source of inefficiency. From the bank capital structure in (3.23), the deposit ratio chosen by bank managers is increasing in the average return on capital (R_t^A) and is decreasing in the deposit rate (R_t). As we will show in Section 4.2, either a positive productivity shock or an expansionary monetary shock would lead to an increase in R_t^A and a decrease in R_t, so bank managers would raise the deposit ratio and issue more deposits, resulting in an amplification effect on those shocks. The amplification effect would generate excessive bank risk and economic fluctuations because a higher level of deposit ratio increases the probability of bank runs. The excessive bank risk and volatility caused by the amplification effect has a negative effect on the welfare and is not taken into consideration by bank managers when they choose their bank capital structure. This is another source of inefficiency in their model.

In short, banks choose a deposit ratio that is higher the socially optimal level because of two reasons: First, they do not fully internalize the negative effect of the early liquidation cost on the resource constraint; Second, they do not take into account the excessive volatility caused by the amplification effect. The next section presents a calibration exercise which shows that the bank capital structure (3.23) achieves a lower welfare than that of banks taking no risk, verifying the over-borrowing feature of the AF model.

3.3.1 The Calibration Exercise

In this section, we solve the steady state of variables in the AF model and compare the welfare with that in the scenario in which bank managers choose a safe capital structure. The welfare criteria are the conditional expected lifetime utility of households to the second order approximation as in Schmitt-Grohe and Uribe (2007).[9] We use

[9] By approximating the utility function to the second order, we take into account the transitional effects from the non-stochastic steady states to stochastic steady states under different bank capital

Table 3.2: Parameter values

	Parameters and descriptions	AF model	Modified model
Hosueholds:			
β	time discount factor	0.995	0.99
σ	risk aversion	2	2
η	utility weight of labour	6.25	6.25
Producers:			
α	capital share	0.3	0.3
δ	physical capital depreciation rate	0.025	0.025
v	elasticity of asset prices to investment	0.5	0.5
ϱ_1	capital adjustment parameter	0.158	0.158
ϱ_2	capital adjustment parameter	-0.025	-0.025
Retailers:			
ϵ	elasticity of substitution	6	6
ϑ	price adjustment cost	30	30
Banks:			
h	dispersion of corporate return	0.35	0.35
c	cost of early liquidation	0.2	0.2
λ	bank manager's liquidation advantage	0.45	1
θ	exogenous survival rate of banks	0.975	0.975
ω	the fraction of initial net worth of new bankers	N/A	0.005
Θ	bank manager fraction of diversion	N/A	0.44
Government:			
G_Y	steady state proportion of government purchase	0.25	0.25
b_π	inflation coefficient of Taylor rule	1.5	1.5
b_Y	output gap coefficient of Taylor rule	0.5	0.5
b_r	lag coefficient of interest rate	0.6	0.6

consumption equivalent to compare the welfare under different bank capital structures. It is the percentage increase (Γ) of household's consumption needed under one bank capital structure to reach the same level of welfare under the 'better' capital structure.

The parameter values used in the AF model are shown in the first column of Table 3.2, and they are specified as follows. The dispersion of return on investment project (h) is chosen to be 0.35 to match the dispersion of US corporate returns in the data. The survival rate of banks (θ) is set to be 0.975 so that banks stay in the banking sector for 10 years on average. The liquidation cost c is set by looking at the recovery rate which is up to $80\% - 90\%$, so $c = 0.2$ is used. The parameter values for other sectors are very conventional. The elasticity of asset prices to investment is $v = 0.5$. The parameter values of $\varrho_1 = 0.158$ and $\varrho_2 = -0.025$ are chosen to achieve the following two targets: First, the steady state asset price Q_t is 1; Second, the steady state investment is the same as that without the adjustment cost.

The bank's demand for deposits is given by (3.23), and the supply of deposits is

structures.

from households as in the Euler equation (3.3). Writing them in the steady-state form gives

$$\bar{d} = \frac{1}{\bar{R}} \frac{\bar{R}^A + h}{2 - \lambda + c(1 + \lambda)} \tag{3.33}$$

and

$$\beta(1 - \bar{\zeta})\bar{R} = 1, \tag{3.34}$$

where

$$\bar{\zeta} = \frac{\bar{R}\bar{d} - \bar{R}^A + h}{2h} \left[1 - \frac{(1 + \lambda)(1 - c)}{4} \left(1 + \frac{\bar{R}^A - h}{\bar{R}\bar{d}} \right) \right]. \tag{3.35}$$

The aggregate bank capital evolve as (3.28) where ϱ_{t+1}^{BK} is the gross return on bank asset. If we combine (3.20), (3.21), and (3.28) in the steady state form to eliminate B_t, we can get

$$1 - \bar{d} = \theta \bar{\varrho}^{BK} = \theta \frac{(\bar{R}^A + h - \bar{R}\bar{d})^2}{8h}. \tag{3.36}$$

By combing (3.34),(3.33), and (3.36), we can solve for the steady state of (d_t, R_t, R_t^A). The other steady state variables can be derived by substituting the above results into the equations in the steady state system.

In the second scenario, in which bank managers choose a safe bank capital structure, the deposit ratio d_t^S is given by:

$$d_t^S = \frac{R_t^A - h}{R_t}. \tag{3.37}$$

To avoid the possibility of bank runs, d_t^S is the highest level of deposit ratio that banks could obtain at time t given the uniform distribution assumption. We call it "safe banking" as in this case, banks can always fully repay the depositors. The steady state values of the relevant variables in both scenarios are shown in the first two sub-columns in Table 3.3. The first sub-column shows the steady state results in the AF model, and the second sub-column is for the safe banking scenario. The parameter values we use are the same as AF. Results show that although the risky bank capital structure in AF achieves higher steady state values of total borrowing, output, and consumption,

Table 3.3: Steady State Values

Variables		AF model		Modified model	
		Risky banking	Safe banking	Risky banking (baseline model)	Safe banking
Return on capital (%)	R^A	22.9%	23.5%	1.5%	1.3%
Return on deposits (%)	R	6%	0.5%	1.1%	0.5%
Deposit ratio	d	0.825	0.781	0.662	0.653
Bank run probability (%)	ϕ	10.6%	0	0.6%	0
Conditional expected loss	g	0.48	0	0.20	0
Total bank assets	QK	0.294	0.280	4.112	4.918
Deposits	D	0.243	0.218	2.721	3.215
Bank capital	B	0.051	0.061	1.391	1.703
Labor	N	0.3	0.295	0.3	0.301
Output	Y	0.298	0.290	0.658	0.6945
Consumption	C	0.211	0.210	0.388	0.398
Payoff to depositors		0.244	0.219	2.749	3.247
Payoff to capitalists		0.065	0.063	1.401	1.721
Payoff to bank managers		0.053	0.063	0	0
Welfare	Γ	1.5%	0	2.6%	0

the safe banking gives higher welfare. It is because of the lower volatility of variables without the risk of bank runs.

Also, it is worth noting that the steady state of return on capital R^A seems very high in both cases (22.9% in the AF model and 23.5% in the safe banking, respectively) which imply unrealistically high quarterly corporate returns. In fact, the high return on capital can be expected by the AF assumptions to manipulate a risky bank leverage and excessively high level of borrowing. Recalling from Table 3.1, bank managers can take half of the proceeds from bank capitalists in no-bank-run case (*Case B*) and get a smaller share of payoff from depositors in bank-run case (*Case A*). In this situation, raising the deposit ratio can reduce the proceeds that bank managers take from bank capitalists and increase the payoff they take from depositors. Thus, a higher level of deposit ratio reduces the return of bank managers and conversely raises the combined return of the outsiders. As we can see from the last several rows of Table 3.3, bank managers take quite a large share of payoff from bank capitalists in the steady state and therefore, the payoff to the bank capitalists is very low. Also, bank capitalists take 2.5 percent ($\theta = 0.975$) of bank capital out of the banking sector in every period. The above facts in the AF model lead to a very low steady-state bank capital and therefore, an unrealistically high return on capital, R^A.

To summarize, the assumptions that bank managers take a large share of proceeds from bank capitalists in the no-bank-run case and that they maximize the combined return of outsiders lead banks to choose a risky bank capital structure which exposes banks to runs. These assumptions generate the inefficiency of over-borrowing in the AF model, but inevitably, they also lead to the level of return on capital that is too high compared to the data. This by-product cannot be eliminated by changing parameter values within a reasonable range.

3.4 Modifying the Banking Sector

To modify the above undesirable feature in the original AF model, we abandon the assumption that bank managers have the advantage in liquidating projects. Instead, we assume that bank capitalists take the role of the bank managers to make balance sheet decisions, so they own and manage banks by themselves. In the situation, bank capitalists take all the proceeds if there is any. Also, we change the assumption that the bank manager maximizes the rate of return to the outsiders combined. In the AF model, this assumption is crucial for banks to take the risk of bank runs and to borrow beyond the socially optimal level, and it is also an important assumption for the existence of equilibrium and tractability. However, this assumption is unreasonable in that the bank managers maximize the rate of return rather than the profit level. By changing the above assumptions, the over-borrowing feature in the AF model also vanishes, and there does not exist an equilibrium. Alternatively, we adopt a banking model from Gertler and Karadi (2011) to remedy these issues. For simplicity, we assume $\lambda = 1$ onward and call the bank capitalist who is both the owner and manager of the bank 'banker'. The following are the alternative assumptions in our modified model.

First, bankers maximize their own payoff only. This assumption in GK brings about a different source of inefficiency: bankers do not fully internalize the cost of

Table 3.4: Payoffs to depositors and bankers in the modified model

	Case A^*	*Case* B^*
Realizations of x_{t+1}	$[-h, R_t d_t - R_t^A]$	$[R_t d_t - R_t^A, h]$
Bank condition	Bank run	No bank run possibility
Depositors	$(1-c)(R_t^A + x_{t+1})Q_t L_t$	$R_t D_t$
Bankers	0	$[(R_t^A + x_{t+1}) - R_t d_t]Q_t L_t$

early liquidation caused by bank runs (if there is any), and it would lead bankers to issue deposits beyond the socially efficient level. Second, a moral hazard problem of banks and depositors is embedded into the model which ensures the existence of the equilibrium and tractability. The above features of the GK model guarantee that the banker's ability to get funds is limited by their internal bank capital. For simplicity and exposition purpose, we keep the notations in Section 2.4. Since the modifications are the assumptions in the banking sector, the bank capital structure (3.23) will change accordingly, but the other equations in the system remain. Depending on the realization of the return of investment projects, there are two different cases which correspond to *Case* A and *Case* $B\&C$ in the AF model.

Case A^*: Bank run happens when the bank cannot fully repay the depositors, i.e., $(R_t^A + x_{t+1})Q_t L_t < R_t D_t$. Depositors run on the bank, so the banker has to liquidate the project early and incurs a liquidation cost. After liquidation, the bank goes bankrupt. The banker exits the banking sector with nothing because he is a mere residual claimant. Depositors get $(1-c)(R_t^A + x_{t+1})Q_t L_t$.

Case B^*: No bank run happens when depositors get the full payment $R_t D_t$, i.e., $(R_t^A + x_{t+1})Q_t L_t > R_t D_t$. In this case, the banker obtains the leftover $(R_t^A + x_{t+1})Q_t s_t - R_t D_t$ and keep it within the bank as bank capital. Table 3.4 summarizes the payoff to depositors and the banker in both *Cases* A^* and B^*.

At the end of each period, if the bank does not go bankrupt, the banker faces a constant probability θ of staying in the banking sector. When bankers are forced to exit the banking sector, they consume all their bank capital. The probability of bankruptcy is the same as ϕ_{t+1} in (3.25). Instead of maximizing the rate of return on assets, the bankers' objective is to maximize the expected discounted terminal consumption when they exit the banking sector:

$$V_t = \max \mathbb{E}_t \sum_{\tau=t+1}^{\infty} \left[(1-\theta)\theta^{\tau-t-1}\Xi_{t,\tau}B_\tau\right], \tag{3.38}$$

where $\Xi_{t,\tau} = \prod_{i=t+1}^{\tau}(1-\phi_i)\Lambda_{t,\tau}$ is the augmented discount factor of bankers.

Bankers care about their own payoff only. As long as the bankers make profit by working as a financial intermediary, they keep accumulating bank capital until they exit the banking sector. When the bank can fully repay depositors at the end of period t, the bank capital accumulation function is given by

$$B_{t+1} = (R_t^A + x_{t+1})Q_t L_t - R_t D_t. \tag{3.39}$$

Following the idea from Gertler and Karadi (2011), we adopt a moral hazard problem in the banking sector: After the banker gets the deposits from households and before lending to the investment projects, he has an outside option to divert a fraction Θ of bank assets to his own family. When the diversion happens, the bank cannot repay the depositors for sure and will go bankrupt. Knowing the possibility of diversion by the banker, depositors limit their funds they lend to the bank to the extent that the fraction of diversion is no more than the expected discounted ultimate bank capital that the bank could get if the banker keeps the bank assets in the bank:

$$V_t \geq \Theta Q_t L_t. \tag{3.40}$$

We can solve the optimization problem of the banker by maximizing (3.38) subject to (3.39) and (3.40). Define the bank leverage as the ratio of bank assets to bank capital:

$$\gamma_t = \frac{Q_t L_t}{B_t} = \frac{1}{1-d_t}. \tag{3.41}$$

The closed form of V_t is given by:

$$V_t(Q_tL_t, B_t) = \mu_t Q_t L_t + \nu_t B_t, \tag{3.42}$$

where the Lagrange multipliers (LMs) in (3.42) are recursively defined as

$$\nu_t = \mathbb{E}_t \Xi_{t,t+1} \Omega_{t+1} R_t, \tag{3.43}$$

$$\mu_t = \mathbb{E}_t \Xi_{t,t+1} \Omega_{t+1} (R_t^A - R_t), \tag{3.44}$$

$$\Omega_{t+1} = 1 - \theta + \theta(\gamma_{t+1}\mu_{t+1} + \nu_{t+1}). \tag{3.45}$$

Above, Ω_{t+1} is the shadow price of the bank capital tomorrow, ν_t is the bank's private cost of issuing deposits (or, the private value of a unit of bank capital), and μ_t is the net profit of bank assets. As the borrowing constraint (3.40) is binding, by (3.41) and (3.42), the optimal bank leverage ratio γ_t is

$$\gamma_t = \frac{\nu_t}{\Theta - \mu_t}, \quad \text{or} \quad d_t = \frac{\nu_t + \mu_t - \Theta}{\nu_t}. \tag{3.46}$$

As mentioned in Section 2.4, the bank assets are lent to firms, so $Q_tL_t = Q_tK_t$. If we consider the evolution of bank capital in aggregate, the dynamics of new and old bankers need to be taken into account. At the end of period t, new bankers enter the banking sector with the initial net worth $\omega R_t^A Q_t K_t$, so the aggregate bank capital B_t evolves as

$$B_{t+1} = \theta \left[\frac{1}{2h} \int_{R_t d_t - R_t^A}^{h} (R_t^A + x_{t+1}) dx_{t+1} Q_t K_t - R_t D_t \right] + \omega R_t^A Q_t K_t. \tag{3.47}$$

By embedding the moral hazard constraint into the banking model, banks face balance sheet constraints endogenously determined by the internal bank capital owned by banks. It also creates a different link between the riskiness of bank assets h and the bank manager's decisions on capital structures through asset prices. When h is high, the probability of bankruptcy increases (see (3.25)), and the net value of bank assets ν_t and μ_t decrease due to a smaller discount factor $\Xi_{t,t+1}$, and banks tend to decrease the borrowing. Hence the increase in h has a negative effect on bank leverage γ_t.

Most of all, bankers tend to borrow more than the socially optimal level as in AF. While the modified model keeps the part that bankers do not take the excessive volatility into consideration, it has a different source of inefficiency which is favorable to excessive borrowing: the bankers maximize their own payoff and fail to internalize the liquidation cost of bank runs. The moral hazard constraint (3.40) is not favorable to excessive borrowing because it limits banks abilities to collect funds, but it guarantees that the bank leverage is finite. Overall, the modified model generates an feature of over-borrowing. In the next section, we illustrate the simulation results.

3.5 Simulation

3.5.1 Steady States

In this section, we compute the steady state of the modified model and show that our model resolves the calibration problem of the AF model mentioned in section 3.3.1. Meanwhile, the model is able to retain the over-borrowing feature of the AF model.

The parameter values we use for the modified model are shown in the second column of Table 3.2, and they are the same as the original AF model except the time discount factor that we use $\beta = 0.99$ for more reasonable steady state values of deposit rate and corporate return.[10] The new parameters introduced in the modified model are as follows: To achieve a reasonable level of bank leverage ratio, we choose the initial net worth of new bankers to be $\omega = 0.5\%$ of the total bank assets and the banker's fraction of diversion to be $\Theta = 0.44$.

As in Section 3.1, we also compare the steady state values of the modified model

[10]We calibrate the return on capital and the deposit return so that the quarterly credit spread is about 30 basis points (the level during the great moderation). If we choose $\beta = 0.995$, either the credit spread or the bank leverage becomes too low. Changing β from 0.995 to 0.99 does not change the main results of the paper.

with those of the safe banking scenario (see (3.37)). The last two sub-columns of Table 3.3 list the steady state values. The first sub-column is under (3.46), and the second sub-column is under (3.37). As we can see from the table, the modified model can get more reasonable values of quarterly return on bank assets 1.5% and the bankruptcy probability 0.6%. And more, the modified model is able to retain the over-borrowing feature of the AF model. Under the safe banking, the steady-state deposit ratio is lower, but the total borrowing level is higher than that in the modified model. It is due to the fact that there is no possibility of bank run in the safe banking, so no liquidation cost is deducted from the resource constraint. Bankers can accumulate more bank capital and therefore, it leads to a higher level of physical capital, output and consumption. As a result, the safe banking gives higher welfare than that in our baseline model.

The fact that the welfare under our baseline model is lower than that under the safe banking scenario implies that bankers borrow more than the socially optimal level. However, the source of inefficiency is very different from that of the AF model. It is the bankers overlooking the liquidation cost caused by bank runs that leads them to borrow too much from households. In such framework, bank regulations which prevent banks from issuing excessive deposits can be welfare improving.

3.5.2 Impulse Response Functions

In this section, we examine the impulse responses of variables to two types of shocks: a positive technological shock (A_t) and an expansionary monetary shock (M_t). We discuss the responses in our modified model as well as in the AF model to investigate the different channels through which banks choose to increase the borrowing level in a boom, resulting in an amplification effect.

We adopt the same setting as the AF model for both shocks. The log-deviation of the total factor productivity $\hat{A}_t$ follows an autoregressive process of

$$\hat{A}_t = \rho_A \hat{A}_{t-1} + \varepsilon_t^A, \tag{3.48}$$

where $\rho_A = 0.95$, ε_t^A is i.i.d. normal with mean 0 and standard deviation 0.056, and the variable with '$\wedge$' is its log-deviation from the steady state. The Taylor-type interest rate rule is subject to a moderately persistent monetary shock M_t where

$$\hat{M}_t = \rho_M \hat{M}_{t-1} + \varepsilon_t^M. \tag{3.49}$$

The autocorrelation coefficient is $\rho_M = 0.2$ and the standard deviation of ε_t^M is 0.006.[11]

The Modified Model

In the modified model, the bank leverage ratio in (3.46) is increasing in the shadow value of bank capital (ν_t) and the net profit of bank assets (μ_t): A higher level of ν_t implies that bank capital becomes more valuable, so bankers can issue more deposits per unit of bank capital. A higher level of μ_t means that banks can get higher profit per unit of bank asset, so it also relaxes the financial constraint and allows banks a higher level of borrowing. Thus, the amplification effect of the banking sector is mainly through the prices of bank capital and assets.

Figure 3.1 shows the responses to a one-standard-deviation monetary expansion in the modified model. A negative shock to $\hat{M}_t$ in the interest rate rule (3.32) causes the nominal interest rate (R_t) to decrease. With sticky prices, it lowers the real interest rate and according to the Euler Equation (3.3), households choose to increase the consumption (C_t), so the demand for final goods in Eq. (3.7) increases. Final good producers choose to raise the prices of their goods, so the inflation rate (π_t) increases. A higher level of output induces investment to increase, and there is a delayed rise in physical capital due to the capital adjustment cost. Because the increase in capital is not as large as that in output and also because its increase is delayed, the rental rate of capital ($\frac{Z_t}{P_t}$) goes up (see Eq. 3.13). Also, the capital adjustment cost generates the pro-cyclicality of asset prices (Q_{t+1}) and according to Eq. (3.17), the price of capital (Q_{t+1}) increases. As a result, a higher level of $\frac{Z_t}{P_t}$ and Q_{t+1} leads to a higher return on

[11] The parameter values for shocks are from the business cycle literature such as Rudebusch (2002).

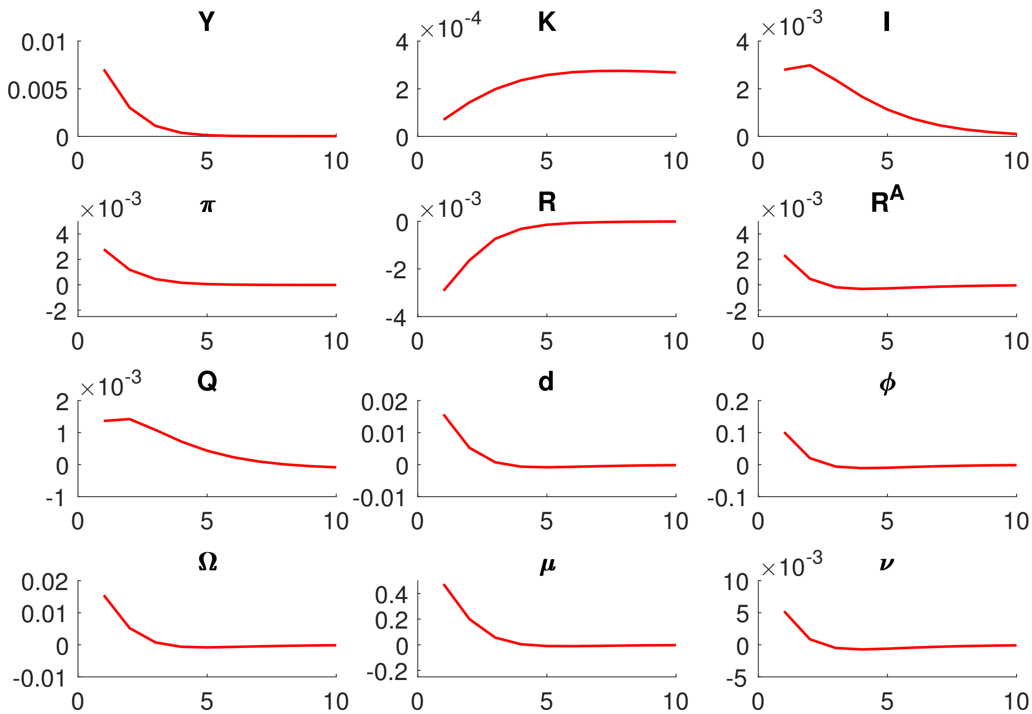

Figure 3.1: Impulse response to a one-standard-deviation monetary expansion in the modified model

capital (R_t^A). The above responses of variables to an expansionary monetary shock can happen in a standard DSGE model.

As is shown in the figure, both the net profit of bank asset (μ_t) and the shadow value of bank capital (ν_t) increase and by (3.46), both effects lead to a higher level of bank leverage. By (3.44), the increase in R_t^A and the decrease in R_t cause the net profit per unit of bank asset μ_t to rise. By (3.45), an increase in μ_t has a large positive effect on the shadow price of the bank capital Ω_t because it is amplified by the bank leverage γ_t. Although a decrease in R_t has a negative effect on ν_t, the overall response of ν_t is positive because the positive effect of Ω_{t+1} on ν_t is larger.[12] In short, an expansionary monetary shock causes the bank leverage γ_t (alternatively, the deposit ratio d_t) to increase, which amplifies the positive effect of the shock on real variables such as output and consumption.

[12] For more details of these mechanism in GK, see Liu (2016)

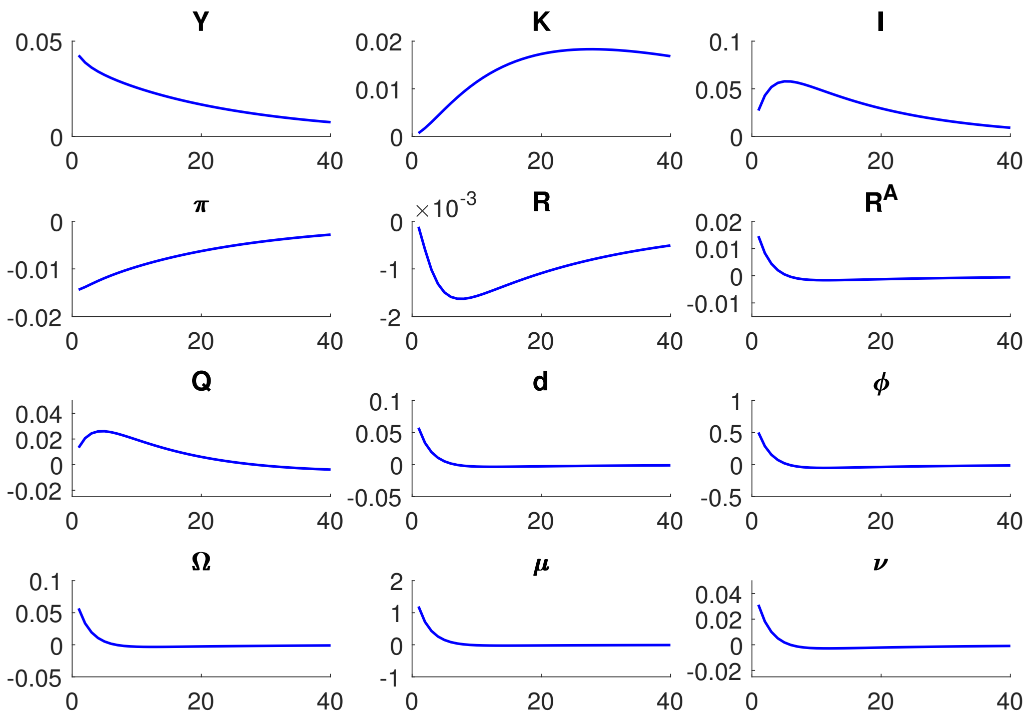

Figure 3.2: Impulse response to a one-standard-deviation productivity increase in the modified model

Figure 3.2 shows the responses to a one-standard-deviation positive productivity shock. The positive productivity shock raises the supply of final goods. To sell more goods, firms lower the prices, so the inflation rate goes down. As the interest rate rule responds largely to the inflation rate, the nominal interest rate drops accordingly. The return on capital is influenced by the following effects: First, an increase in the total factor productivity raises the marginal product of capital, so the rental rate of capital (Z_t/P_t) increases and it has a positive effect on R_t^A; Second, due to an increase in output and then investment, the asset price (Q_{t+1}) goes up and it raises R_t^A; Third, the increased cost of capital adjustment dampens the increase in R_t^A. In general, the overall effect on R_t^A is positive. A higher level of R_t^A and a lower level of R_t causes the net profit of bank assets (μ_t) to increase. Similar to the expansionary monetary shock, the private value of bank capital ν_t increases because of the positive response of Ω_{t+1}. All of these effects lead to an amplification effect on a productivity shock via the shadow values of bank asset and bank capital and bank leverage.

The AF Model

In the modified model, banks tend to raise their leverage after a positive shock because the responses of R_t^A and R_t to shocks raises the shadow values of bank asset and bank capital, which relax the borrowing constraint. Unlike our modified model, the amplification channel is directly through the responses of R_t and R_t^A to shocks in the AF model. According to the bank capital structure (3.23) in AF, the bank leverage ratio is decreasing in the deposit rate (R_t) and is increasing in the return on capital (R_t^A). Since the modified model is the same as the AF model except for the banking sector, the responses of variables to shock in other sectors in the AF model are similar. When there is an expansionary monetary shock, for example, inflation and output goes up, interest rate goes down, and the return on capital goes up due to the presence of capital adjustment cost. In this situation, the cost of issuing deposits (R_t) is lower and the average return of bank asset (R_t^A) is higher. By (3.23), the bank chooses a higher level of deposit ratio which causes an amplification effect. The similar idea applies to a positive productivity shock.

In both the AF model and our modified model, the amplification effect induces banks to borrow more after positive shocks, and it leads to a higher probability of bank runs. In the modified model, a higher bank risk implies a higher overall liquidation cost which has not been taken into consideration when bankers choose the leverage ratio. It also causes excessive volatility and harms the welfare. That is to say, a countercyclical macroprudential policy which controls the bank leverage in good times and encourages deposit issuance in bad times can have a stabilizing effect on the economy and improve the welfare.

3.6 Concluding Remarks

Understanding the source of inefficiency in a macro model is as important as the quantitative results that it can generate. By making the source of inefficiency clear, we can explain why some policies can counteract the inefficiency and improve welfare. This paper identifies the source of inefficiency in Angeloni and Faia (2013) and demonstrates that some undesirable features of the calibration results of their model stem from their somewhat unnatural way of introducing inefficiency. This calls for an alternative way of modeling banks' over-borrowing feature in a DSGE model with risky banking system.

To achieve that, this paper embeds a banking model from Gertler and Karadi (2011) into the AF model. Like Angeloni and Faia (2013), our modified model generates the following features that are crucial for macroprudential policies to play a role: First, banks take excessive leverage ratio relative to the socially optimal level. Second, banks tend to borrow more when the economy is in a boom than in recession, leading to amplification of macroeconomic shocks. The difference is, however, the way the model creates inefficiency. In our modified model, the inefficiency is that bankers consider only their own profit and do not internalize the potential cost of early liquidation on the resources in the economy. The modified model improves the calibration of the AF model.

Based on that, one further research topic is to find the optimal macroprudential policy that has the best way to counteract the over-borrowing feature and the excessive volatility due to the amplification effect.

Appendix

3.6.1 Modified Model: System of Equations

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{3.50}$$

$$\left[1 - \frac{\vartheta}{2}\left(\pi_t - 1\right)^2\right] Y_t - \Delta_t = C_t + I_t + G_t. \tag{3.51}$$

$$\Delta_t = \frac{c}{4h} Q_t K_t \left[(R_t d_t)^2 - (R_t^A - h)^2\right]. \tag{3.52}$$

$$K_{t+1} = (1-\delta)K_t + \xi\left(\frac{I_t}{K_t}\right) K_t, \tag{3.53}$$

$$\xi'(\frac{I_t}{K_t}) Q_{t+1} = 1. \tag{3.54}$$

$$\frac{R_t^A}{\pi_{t+1}} = \frac{\alpha m_t A_t \left(\frac{N_t}{K_t}\right)^{1-\alpha} + Q_{t+1}\left[(1-\delta) - \xi'(\frac{I_t}{K_t})\frac{I_t}{K_t} + \xi(\frac{I_t}{K_t})\right]}{Q_t}. \tag{3.55}$$

$$\mathbb{E}_t\left[\Lambda_{t,t+1}(1-\zeta_{t+1})\right] R_t = 1, \tag{3.56}$$

$$\Lambda_{t,t+1} = \beta \frac{C_t^{\sigma}}{C_{t+1}^{\sigma}}. \tag{3.57}$$

$$\zeta_{t+1} = (R_t d_t - R_t^A + h)\left[\frac{1+c}{4h} - \frac{1-c}{4h}\frac{R_t^A - h}{R_t d_t}\right]. \tag{3.58}$$

$$(1-\alpha) m_t \frac{Y_t}{N_t} = \frac{\eta C_t^{\sigma}}{1 - N_t}. \tag{3.59}$$

$$Q_t K_t = D_t + B_t; \tag{3.60}$$

$$\gamma_t = \frac{\nu_t}{\Theta - \mu_t}, \tag{3.61}$$

$$\Xi_{t,t+1} = (1 - \phi_{t+1})\Lambda_{t,t+1}. \tag{3.62}$$

$$\phi_{t+1} = \frac{R_t^A + h - R_t d_t}{2h}. \tag{3.63}$$

$$\mu_t = \Xi_{t,t+1}\Omega_{t+1}(R_t^A - R_t), \tag{3.64}$$

$$\nu_t = \Xi_{t,t+1}\Omega_{t+1} R_t, \tag{3.65}$$

$$\Omega_{t+1} = 1 - \theta + \theta(\gamma_{t+1}\mu_{t+1} + \nu_{t+1}). \tag{3.66}$$

$$B_t = \theta \left[\frac{1}{4h} (R_t^A + h + R_t d_{t-1})(R_t^A + h - R_t d_{t-1}) Q_{t-1} K_{t-1} - R_t D_{t-1} \right] + \omega R_t^A Q_{t-1} K_{t-1}. \tag{3.67}$$

$$\gamma_t = \frac{Q_t K_t}{B_t}. \tag{3.68}$$

$$d_t = \frac{D_t}{Q_t K_t}. \tag{3.69}$$

$$(\pi_t - 1)\pi_t = \frac{\epsilon}{\vartheta}(m_t - \frac{\epsilon - 1}{\epsilon}) Y_t + \mathbb{E}_t \left[\Lambda_{t,t+1} (\pi_{t+1} - 1)\pi_{t+1} \right]. \tag{3.70}$$

$$\ln \left(\frac{R_t}{R} \right) = (1 - b_r) \left[b_\pi \ln \left(\frac{\pi_t}{\pi} \right) + b_Y \ln \left(\frac{Y_t}{Y} \right) \right] + b_r \ln \left(\frac{R_{t-1}}{R} \right) + \hat{M}_t. \tag{3.71}$$

3.6.2 Maximization Problem of Good Producers

Final good producers choose $(P_t(i), K_t(i), N_t(i))$ to maximize (3.10) subject to (3.5) and (3.7). The Lagrangian is

$$\begin{aligned} \mathcal{L}_1 &= \mathbb{E}_t \sum_{\tau=t}^{+\infty} \Lambda_{t,\tau} \Bigg\{ \frac{P_\tau(i)}{P_\tau} Y_\tau(i) - \frac{W_\tau}{P_\tau} N_\tau(i) - \frac{Z_\tau}{P_\tau} K_\tau(i) - \frac{\vartheta}{2} \left(\frac{P_\tau(i)}{P_{\tau-1}(i)} - 1 \right)^2 \\ &\quad - m_\tau \left[\left(\frac{P_\tau(i)}{P_\tau} \right)^{-\epsilon} Y_\tau - A_\tau K_\tau(i)^\alpha N_\tau(i)^{1-\alpha} \right] \Bigg\}, \end{aligned} \tag{3.72}$$

where m_t is the Lagrange multiplier. The first order conditions with respect to (w.r.t.) $K_t(i)$ and $N_t(i)$ are

$$\frac{\partial \mathcal{L}_1}{\partial K_t(i)} = -\frac{Z_t}{P_t} + \alpha m_t A_t K_t(i)^{\alpha-1} N_t(i)^{1-\alpha} = 0, \tag{3.73}$$

$$\frac{\partial \mathcal{L}_1}{\partial N_t(i)} = -\frac{W_t}{P_t} + (1-\alpha) m_t A_t K_t(i)^\alpha N_t(i)^{-\alpha} = 0, \tag{3.74}$$

After simplification, we can derive (3.13) and (3.12). The first order condition w.r.t. $P_t(i)$ is

$$\begin{aligned} \frac{\partial \mathcal{L}_1}{\partial P_t(i)} &= (1-\epsilon) \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_t} - \vartheta \left(\frac{P_t(i)}{P_{t-1}(i)} - 1 \right) \frac{1}{P_{t-1}(i)} + \epsilon m_t \left(\frac{P_t(i)}{P_t} \right)^{-\epsilon-1} \frac{Y_t}{P_t} \\ &\quad + \vartheta \Lambda_{t,t+1} \left(\frac{P_{t+1}(i)}{P_t(i)} - 1 \right) \frac{P_{t+1}(i)}{P_t^2(i)}. \end{aligned} \tag{3.75}$$

Simplifying (3.75) by $P_t(i) = P_t$ and $\pi_t = \frac{P_t}{P_{t-1}}$, we can get (3.14).

3.6.3 Maximization Problem of Capital Producers

Capital producers choose I_t to maximize (3.16) subject to (3.15).The Lagrangian is

$$\mathcal{L}_2 = \mathbb{E}_t \sum_{t=\tau}^{+\infty} \Lambda_{t,\tau} \left\{ \frac{Z_\tau}{P_\tau} K_\tau - I_\tau + Q_{\tau+1} \left[(1-\delta) K_\tau + \xi \left(\frac{I_\tau}{K_\tau} \right) K_\tau - K_{\tau+1} \right] \right\}, \quad (3.76)$$

The first order conditions w.r.t. I_t and K_{t+1} give (3.17) and (3.18).

3.6.4 Maximizing the Total Payoff to Everyone

In this section, we try to find the optimal capital structure d_t if the bank manager maximizes the payoff level to bank capitalists, depositors, and bank managers in the AF model. The objective of the bank manager becomes:

$$\frac{1}{2h} \int_{-h}^{R_t d_t - R_t^A} (1-c)(R_t^A + x_{t+1}) Q_t L_t dx_{t+1} + \frac{1}{2h} \int_{R_t d_t - R_t^A}^{h} (R_t^A + x_{t+1}) Q_t L_t dx_{t+1}. \quad (3.77)$$

Given the bank capital B_t in the bank, the total bank asset can be expressed as $Q_t L_t = \frac{1}{1-d_t} B_t$. Substituting in into the objective yields

$$\frac{1}{2h} \left[\int_{-h}^{R_t d_t - R_t^A} (1-c)(R_t^A + x_{t+1}) dx_{t+1} + \int_{R_t d_t - R_t^A}^{h} (R_t^A + x_{t+1}) dx_{t+1} \right] \frac{1}{1-d_t} B_t. \quad (3.78)$$

We divide the space of d_t into the following different intervals:

(1). When $R_t d_t - R_t^A < -h$, i.e., $0 < d_t < \frac{R_t^A - h}{R_t}$, the objective function becomes:

$$B_t \frac{1}{2h} \left[\int_{-h}^{h} (R_t^A + x_{t+1}) dx_{t+1} \right] \frac{1}{1-d_t}. \quad (3.79)$$

It is an increasing function of d_t, as $B_t \frac{1}{2h} \left[\int_{-h}^{h} (R_t^A + x_{t+1}) dx_{t+1} \right]$ is not a function of d_t and $\frac{1}{1-d_t}$ is increasing in d_t.

(2). When $-h < R_t d_t - R_t^A < h$, i.e., $\frac{R_t^A - h}{R_t} < d_t < 1$[13], the objective function is shown in Eq. (3.77). We can simplify it as follows:

$$\begin{aligned}
& \frac{B_t}{2h}\left[\int_{-h}^{h}(R_t^A + x_{t+1})dx_{t+1} - c\int_{-h}^{R_t d_t - R_t^A}(R_t^A + x_{t+1})dx_{t+1}\right]\frac{1}{1-d_t} \\
= & \frac{B_t}{2h}\left[2hR_t^A - cR_t^A(R_t d_t - R_t^A + h) - \frac{c}{2}[(R_t d_t - R_t^A)^2 - h^2]\right]\frac{1}{1-d_t} \\
= & \frac{B_t}{2h}\left[\frac{c}{2}h^2 + (2-c)hR_t^A + \frac{c}{2}(R_t^A)^2 - \frac{c}{2}R_t^2 d_t^2\right]\frac{1}{1-d_t}. \qquad (3.80)
\end{aligned}$$

Differentiate it with respect to d_t yields

$$\frac{B_t}{2h(1-d_t)^2}\left[-cR_t^2 d_t(1-d_t) + \frac{c}{2}h^2 + (2-c)hR_t^A + \frac{c}{2}(R_t^A)^2 - \frac{c}{2}R_t^2 d_t^2\right].$$

Further,

$$\begin{aligned}
& \frac{c}{2}R_t^2 d_t^2 - cR_t^2 d_t + \frac{c}{2}h^2 + (2-c)hR_t^A + \frac{c}{2}(R_t^A)^2 \\
= & \frac{c}{2}R_t^2(1-d_t)^2 + \frac{c}{2}h^2 + (2-c)hR_t^A + \frac{c}{2}(R_t^A)^2 - \frac{c}{2}R_t^2 > 0. \qquad (3.81)
\end{aligned}$$

We can see that the above first derivative is positive when $R_t^A > R_t$. To sum up, the case (1) and (2) show that banks would fancy higher deposit ratio when their objective is to maximize the payoff of bank capitalists, depositors and bank managers, and it does not have an interior solution.

3.6.5 Maximization Problem of Bankers in the Modified Model

Bankers maximize (3.38) subject to bank capital accumulation (3.39) and the moral hazard constraint (3.40). First, we rewrite (3.39) as:

$$f(B_t, B_{t-1}) = [(R_t^A + x_t - R_t)\gamma_{t-1} + R_t]B_{t-1} - B_t = 0, \qquad (3.82)$$

[13]We know that $\frac{R_t^A + h}{R_t} > 1$, and we don't consider the case when $d_t > 1$.

The Lagrangian is constructed as

$$\mathcal{L}_3 = \mathbb{E}_t \left\{ \sum_{\tau=t+1}^{\infty} (1-\theta)\theta^{\tau-t-1} \Xi_{t,\tau} \left[B_\tau + \Omega_\tau f(B_t, B_{t-1}) \right] \right\}. \tag{3.83}$$

The first order condition with respect to B_{t+1} is:

$$\mathbb{E}_t \Omega_{t+1} = 1 - \theta + \theta \mathbb{E}_t \Xi_{t+1,t+2} \Omega_{t+2} [(R^A_{t+2} - R_{t+2})\gamma_{t+1} + R_{t+2}]. \tag{3.84}$$

Changing the notations as follows:

$$\mu_t = \mathbb{E}_t \Xi_{t,t+1} \Omega_{t+1} (R^A_t - R_t), \tag{3.85}$$

$$\nu_t = \mathbb{E}_t \Xi_{t,t+1} \Omega_{t+1} R_t, \tag{3.86}$$

the FOC (3.84) becomes

$$\Omega_{t+1} = 1 - \theta + \theta(\gamma_{t+1}\mu_{t+1} + \nu_{t+1}). \tag{3.87}$$

The value function of bankers is

$$\begin{aligned} V_t &= \mathbb{E}_t(\Xi_{t,t+1}, \Omega_{t+1}, B_{t+1}), \\ &= \mathbb{E}_t \Xi_{t,t+1} \Omega_{t+1} \left[(R^A_t - R_t)\gamma_t + R_t \right] B_t, \\ &= (\gamma_t \mu_t + \nu_t) B_t. \end{aligned} \tag{3.88}$$

Given the value function form, we can incorporate the the moral hazard constraint:

$$\mathcal{L}_3 = V_t(n_t) + \lambda_t \left[V_t - \Theta Q_t L_t \right]. \tag{3.89}$$

The FOC with respect to λ_t is:

$$\gamma_t = \frac{\nu_t}{\Theta - \mu_t}. \tag{3.90}$$

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